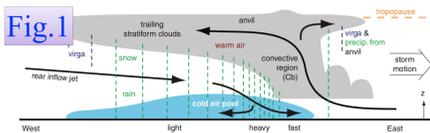


# Identifying Tropical Cold Pools with Singularity Exponents



## 1. Introduction

Tropical cold pools are near-surface regions of cool, dense air formed by downdrafts from deep convection (Fig.1). They play a crucial role in organizing tropical convection by triggering new storms. Their dynamics influence rainfall patterns, cloud formation, and large-scale atmospheric circulations.



When heavy precipitation falls, it is accompanied by evaporatively-cooled air that, on reaching the surface, creates a gust front that expands horizontally in all directions, resulting in local air temperature and moisture fronts that alter the surrounding environment. Gust fronts generate extreme values in probability distributions for velocity gradients (look ahead to Fig.3).

Garg *et al.* [1] developed an objective method to identify wind gradient features and associated them to oceanic cold pools. Their method identifies pixel clusters where the magnitude of the local velocity gradient tensor,  $\nabla \mathbf{u}$ , exceeds a pre-determined threshold. We interpret their threshold as the value distinguishing the set of extreme values.

An alternative approach makes use of singularity exponents. Singularity exponents (first introduced in the multifractal theory of turbulence – see Ref [2]) measure the local regularity of the wind field. They are used in quality control and to identify fronts in wind fields and ocean currents. Importantly, they are estimated from the ASCAT wind fields by the NWP SAF Wind Data Processor (AWDP).

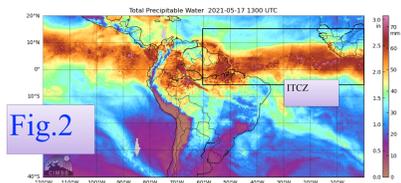
## 2. Objective

Our objective is to show that singularity exponents can be used instead of wind gradients for identification of Tropical cold pools. Essentially features in satellite imagery are identified as clusters of extreme values. In the present context this requires correlating extremes of different quantities. This is achieved here by calculating the Odds Ratio.

## 3. Data

ASCAT winds (12.5 km) from tandem mission, collocated with MSG rain rates (samples every 15 mins at 3 km resolution).

Study area : Atlantic ITCZ (Fig. 2).



## 4. Methods

### A. Wind gradient extremes

The Frobenius norm of the wind gradient tensor,  $|\nabla \mathbf{u}|_F$

$$\nabla \mathbf{u} = \begin{pmatrix} \partial u / \partial x & \partial u / \partial y \\ \partial v / \partial x & \partial v / \partial y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \delta + s_n & -\zeta + s_s \\ \zeta + s_s & \delta - s_n \end{pmatrix}$$

$$|\nabla \mathbf{u}|_F = \sqrt{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2}$$

$$= \frac{1}{\sqrt{2}} \sqrt{\delta^2 + \zeta^2 + s_n^2 + s_s^2}$$

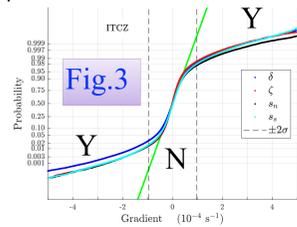


Fig 3: Normal probability plot of wind gradients.

$|\nabla \mathbf{u}|_F > 2\sigma \Rightarrow$  extreme value.

### D. Correlating extremes

Correlating the extremes of  $|\nabla \mathbf{u}|_F, h, R_{max}$  is carried out by binning WVCs in a 2 by 2 contingency table and computing the Odds Ratio,  $OR(\cdot, \cdot)$ .

Significant correlation when  $OR(A, B) > 1$

### B. Singularity exponent extremes

In the neighbourhood,  $r$ , of the point  $\mathbf{x}$ :

$$|\nabla \mathbf{u}(\mathbf{x})| \sim r^{h(\mathbf{x})}$$

$h$  is the singularity exponent.

$h < -0.1 \Rightarrow$  steep gradient (extreme value).

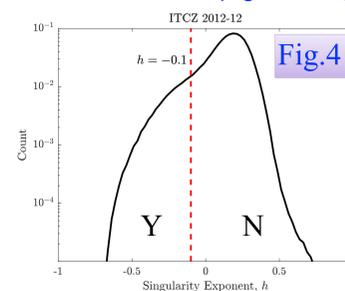


Fig. 4: singularity spectrum for Dec 2012 over the Atlantic ITCZ .

### C. Rain rate extremes

MSG rain rates were regridded to wind vector cells (WVC) with the local maximum 3 km rain rate ( $R_{max}$ ). If  $R_{max} > 3$  m/hr, rain classified as extreme value.

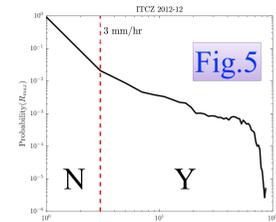


Fig 5: Probability distribution of  $R_{max}$  for Atlantic ITCZ (Dec 2012).

Contingency table for binary variables A and B. Y  $\Rightarrow$  exceeds threshold.

		B	
		Y	N
A	Y	$n_{11}$	$n_{12}$
	N	$n_{21}$	$n_{22}$

$$\text{Odds Ratio} = \frac{n_{11} n_{22}}{n_{12} n_{21}}$$

## 5. Results

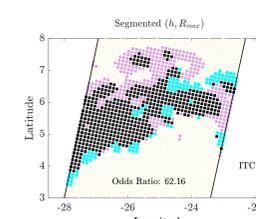
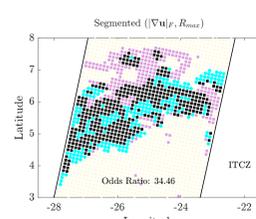
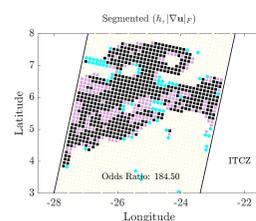
The left column of figs shows spatial correlations for the case indicated in the right column. The correlation is particularly strong between extremes of  $h$  and  $|\nabla \mathbf{u}|_F$ . This is expected, but it is not perfect due to different methods of calculation and selection of thresholds.

The lower two sets of figs indicate extremes for both  $h$  and  $|\nabla \mathbf{u}|_F$  are strongly and similarly correlated with extremes of  $R_{max}$ .

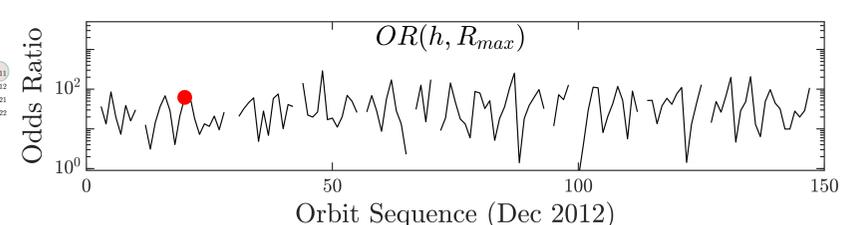
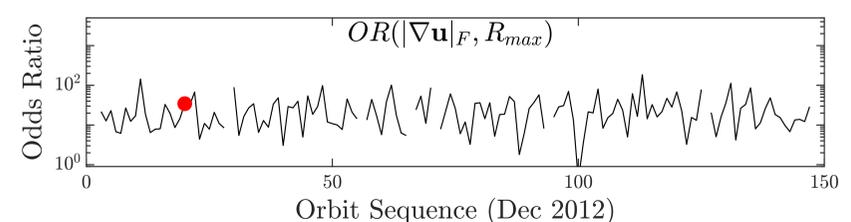
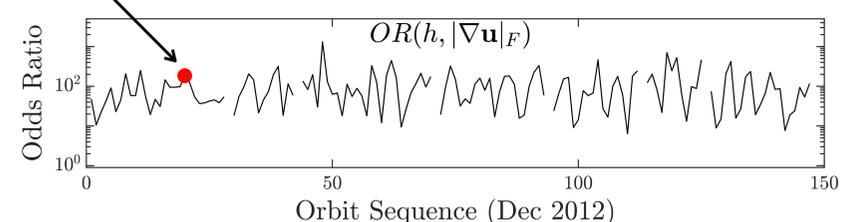
Fluctuations in the Odds Ratio indicate a dependence on the time of day.

### Case showing spatial locations of

$$\{n_{11}, n_{12}, n_{21}, n_{22}\}$$



### Odds Ratio Time sequence



## 6. Conclusion:

The results support the proposal to use the singularity exponents in the ASCAT winds product to identify tropical cold pools. Further work is required to tune thresholds and make further use of singularity exponents to identify features in the wind field and their interaction with other geophysical parameters such as sea surface temperature gradients.

## References

- [1] Piyush Garg *et al.* "Identifying and Characterizing Tropical Oceanic Mesoscale Cold Pools using Spaceborne Scatterometer Winds". In: 125.5 (2020).  
 [2] J. Isern-Fontanet and A. Turiel. On the connection between intermittency and dissipation in ocean turbulence: A multifractal approach. *Journal of Physical Oceanography*, 51(8):2639 – 2653, 2021.