A Theory To Explain Tropical Cyclone Kinetic Energy Spectra

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1. Objective

To develop a theory of TC turbulence that successfully explains observations of ***1D KE energy spectral*** slopes and amplitudes.

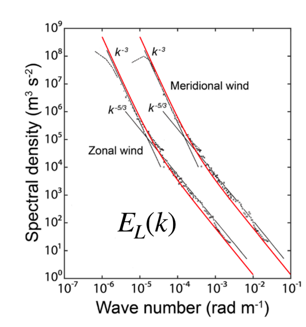
2. Flight Data

- ***Dataset***: Consisted of 3000 reconnaissance aircraft missions into 320 tropical storms and tropical cyclones in the North Atlantic and Eastern Pacific Oceans during the years 1977-2022. Also, NOAA Best Track (NBT) and Hurricane Research Division (HRD) storm parameters and track information.
- ***Samples***: Only flights along *radial legs* were used. The flight legs spanned the full TC. From these, *sublegs* were extracted that sampled either the TC inner core (IR) or TC outer region (OR).
- ***Storm classification***: This was carried out by classifying the maximum 60-second sustained wind speed of each flight leg and grouping them according to the Saffir-Simpson scale.
- ***Wind field regions***: Inner region IR: $0 < r < 2R_{mw}$ and outer region OR: $r > 2R_{mw}$. The IR includes the *eyewall* (green) and the *near-core* (pink). See Fig 1.

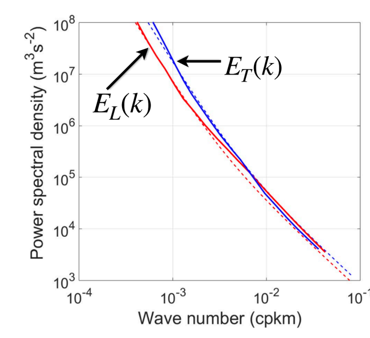
3. Previous Work

Observed atmospheric KE spectra (midlatitudes)

Nastrom & Gage



MOZAIC



Quasi-Normal Scale Elimination (QNSE) ***theory***
See [1]

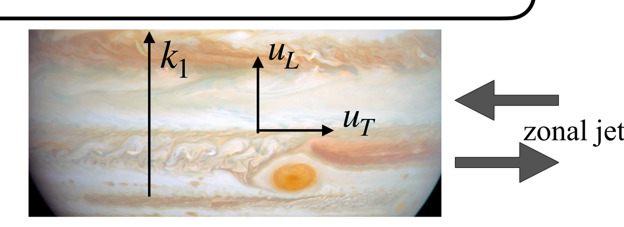
Navier-Stokes rotating frame — Coriolis parameter, f → *Langevin Eqns (stochastic forcing)* ← Apply successive coarse-graining → Predicted isotropic 1D KE spectra

$$E_L(k_1) = C_{KL}\Pi_\epsilon^{2/3}k_1^{-5/3} + 0.0926 f^2 k_1^{-3}$$
$$E_T(k_1) = C_{KT}\Pi_\epsilon^{2/3}k_1^{-5/3} + 0.24 f^2 k_1^{-3}$$

Note…
dependence on f
peristrophic regime
(peristrophy = rotation)

Simulations of 2D turbulence
β-plane; surface of rotating sphere …
- Rossby waves
- Zonal jets

→ Simulations produced *anisotropic* 1D KE spectra

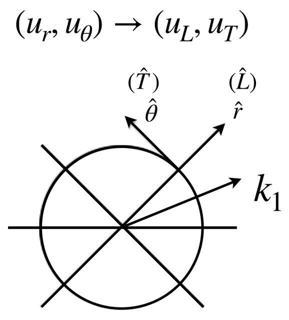


$$E_L(k_1) = C_{KL}\Pi_\epsilon^{2/3}k_1^{-5/3}$$
$$E_T(k_1) = C_{KT}\Pi_\epsilon^{2/3}k_1^{-5/3} + C_z\beta^2 k_1^{-5}$$

($k_1 \perp$ to axis of zonal jet)

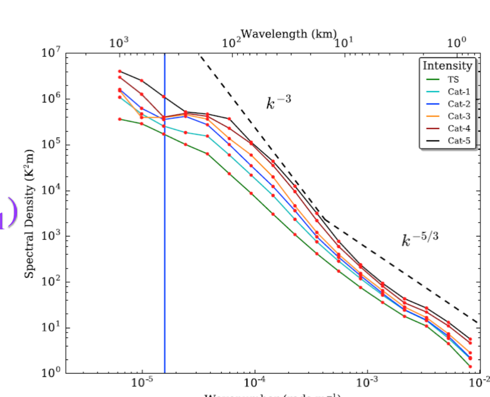
Note…
dependence on β
k_1^{-5} spectra
zonostrophic regime

4. Tropical Cyclones — Previously observed spectra



Vonich & Hakim (2018)
Spectral composites of three datasets. Radial flight legs span TC. Taken from Ref [2].

- flight legs grouped into classes defined by TC category.
- spectra ensemble averaged for each class.



Results …
As TC intensity increases …
- amplitude increases
- slopes steepen
- resemble Nastrom & Gage spectra

5. TC Model

- Barotropic framework
- Rapidly spinning vortex ($\hat{f}$) in a planetary flow (f) …
  - ➡ $\hat{f}$: *cyclostrophic* Coriolis parameter
  - ➡ $\tilde{f}$: *effective* Coriolis parameter

$$\tilde{f} = f + \hat{f}$$

Principle of Least Action
(Landau & Lifshitz: *Mechanics*)

If $\tilde{f}$ = constant in the TC core, theory predicts …

$$E_L(k_1) = C_{KL}\Pi_\epsilon^{2/3}k_1^{-5/3} + 0.0926\tilde{f}^2 k_1^{-3}$$
$$E_T(k_1) = C_{KT}\Pi_\epsilon^{2/3}k_1^{-5/3} + 0.24\tilde{f}^2 k_1^{-3}$$

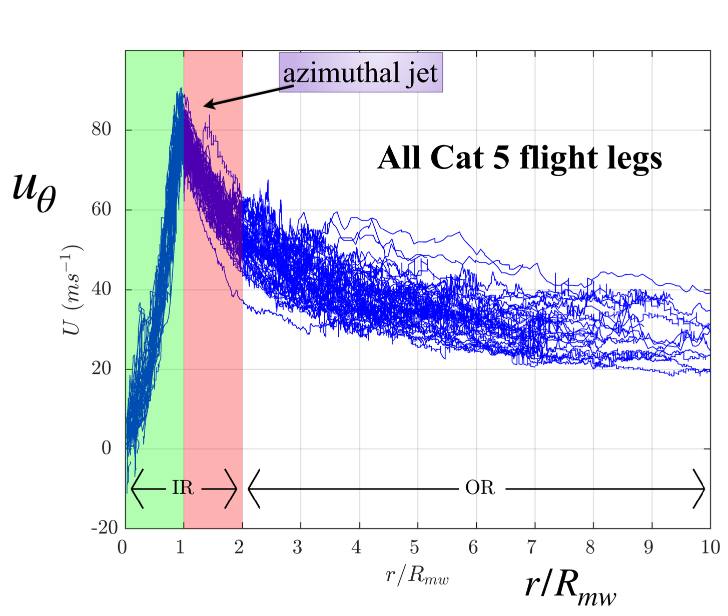
If $\tilde{f} \neq$ constant, there is a ***cyclostrophic*** ***β*-effect** — which generates Vortex Rossby Waves (**VRWs**) in the TC eyewall. $(\hat{\beta} \approx \hat{f}/R_{mw})$

VRWs interact with turbulence and produce *anisotropic* 1D KE spectra …

$$E_L(k_1) = C_{KL}\Pi_\epsilon^{2/3}k_1^{-5/3} + C_{fL}\tilde{f}^2 k_1^{-3}$$
$$E_T(k_1) = C_{KT}\Pi_\epsilon^{2/3}k_1^{-5/3} + C_{fT}\tilde{f}^2 k_1^{-3} + C_z\hat{\beta}^2 k^{-5}$$

6. Comparison with Observations

Structure of the TC wind field



IR
Inner Region - includes eyewall and near-core (pink)

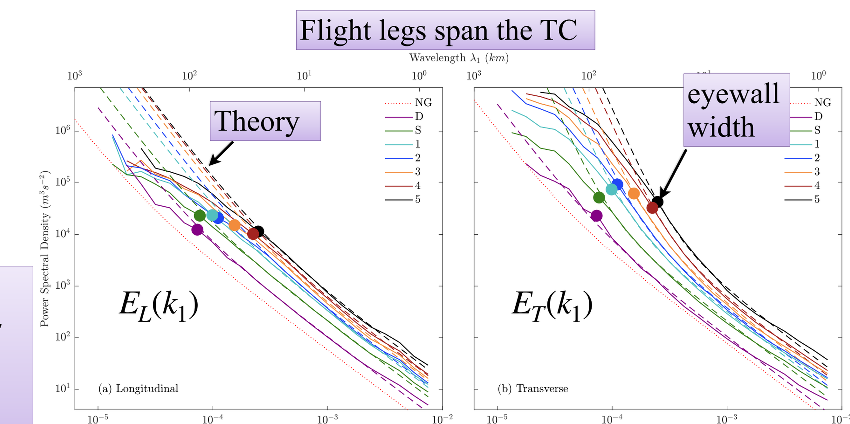
OR
Outer Region

Eyewall azimuthal velocity profile resembles an eastward zonal jet found on Saturn and Jupiter!

Flight legs span the TC

Amplitudes increase with TC category

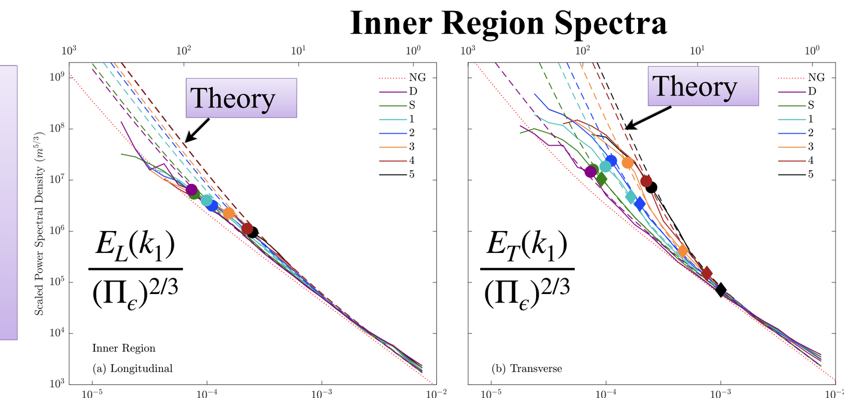
Fig shows that both E_L and E_T are needed to account for spectral anisotropy



Inner Region Spectra

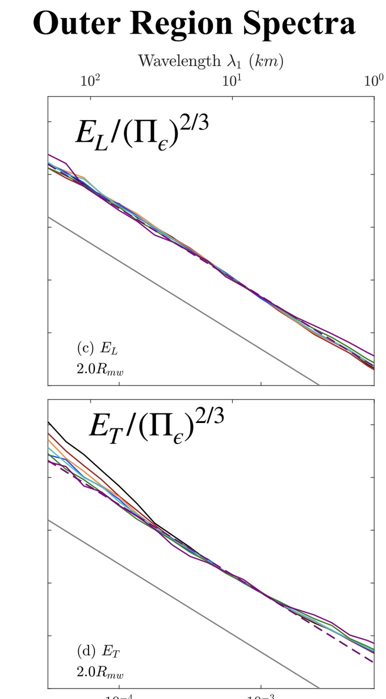
Normalization collapses all spectra in the Kolmogorov subrange.

The subrange shrinks with increasing TC intensity.

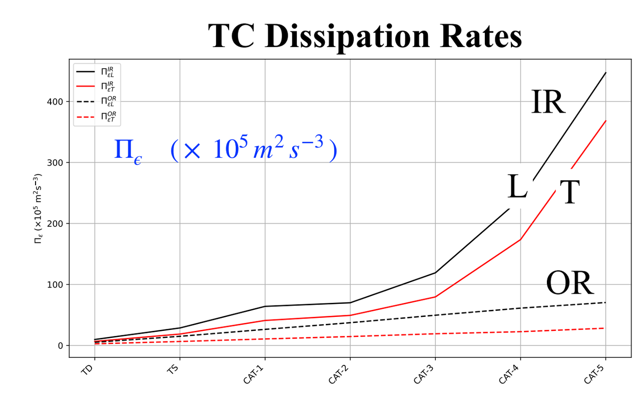


The diamonds mark the boundary between peristrophic and zonostrophic subranges (see Ref [3]).

Outer Region Spectra



TC Dissipation Rates



6. Conclusion:

Observed TC spectra shows that cyclostrophic turbulence evolves from purely peristrophic to mixed peristrophic-zonostrophic turbulence. See Ref [3] for more results and conclusions.

References

[1] Sukoriansky, S., & Galperin, B. (2016). QNSE theory of turbulence anisotropization and onset of the inverse energy cascade by solid body rotation. *J. Fluid Mech.*, **805**, 384–421.
[2] Vonich, P. T., & Hakim, G. J. (2018). Hurricane kinetic energy spectra from in situ aircraft observations. *J. Atmos. Sci.*, **75** , 2523-2532.
[3] Galperin, Nickerson, King and Zhang (2025).