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Quadruple collocation analysis of buoy, scatterometer, and NWP winds

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Introduction

- Triple collocation analysis is an established technique for calculating linear intercalibration coefficients and observation error variances when collocated (in space and time) measurements from three different systems are available, using a simple error model.
- Assumptions error model: (1) linear calibration is sufficient, (2) errors are independent of the true value, and (3) error correlations are known or uncorrelated.
- We extend the formalism to quadruple and higher order collocations and apply it to quadruple collocations of buoys, ASCAT-A/B, ScatSat, and ECMWF forecasts.
- Paper submitted to JGR, preprint available on ESSOAr, doi 10.1002/essoar.10505872.1; see the preprint for more detailed information.

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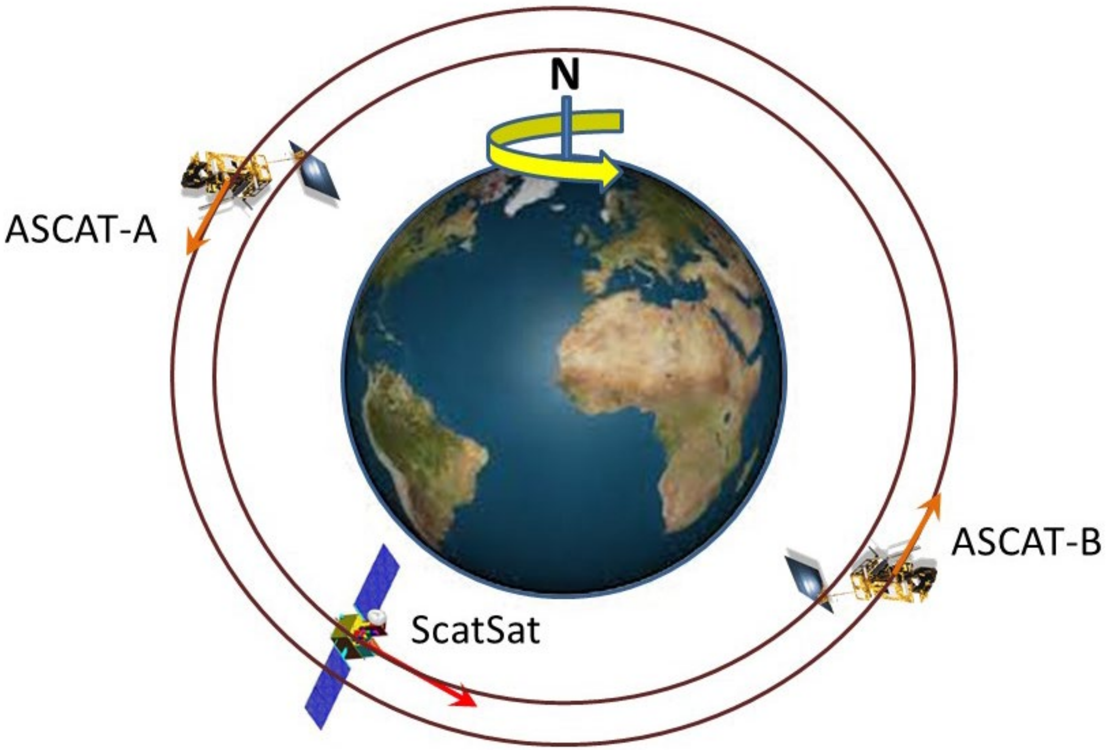
Motivation

- ScatSat was launched by ISRO in September 2016 in the same orbital plane as ASCAT-A and ASCAT-B, but in a slightly lower orbit (see figure).
- As a consequence, it underpasses ASCAT-A and ASCAT-B about twice a week, resulting in a considerable number of buoy-ASCAT-ScatSat-ECMWF collocations.
- Study period 06-10-2016 to 22-07-2017, ScatSat data from ISRO, version 1.1.3 L2A data, processed at KNMI with PenWP.
- Also buoy-ASCAT-A/B-ECMWF collocations.



ASCAT-ScatSat orbital geometry “from above”

The satellites fly over the poles, all in the same orbital plane, while the Earth rotates underneath them

Collocation model (1)

- Suppose that n observation systems make collocated observations in time and space ($n \geq 3$).
- Suppose that linear intercalibration is sufficient. Then the measurements x_i made by observation system i ($i = 1,2, \dots, n$) can be described as

$$x_i = a_i(t + \varepsilon_i) + b_i$$

with a_i the calibration scaling, b_i the calibration bias, t the signal common to all systems (“true signal”) and ε_i a random error with zero average and variance σ_i^2.

- Take system 1 as calibration reference, so $a_1 = 1$ and $b_1 = 0$.

Collocation model (2)

- Now take first moments (averages) over all observations, denoted by the brackets $\langle\ \rangle$ and set $M_i = \langle x_i \rangle$.
- From this one finds the calibration biases as $b_i = M_i - a_i M_1$ and the average of the common signal $\langle t \rangle = M_1$. So once the calibration scalings a_i are found, the calibration biases are known.
- Form second moments $M_{ij} = \langle x_i x_j \rangle$ and use that $\langle t\varepsilon_i \rangle = 0$. Forming covariances $C_{ij} = M_{ij} - M_i M_j$, introduce the common variance $T = \langle t^2 \rangle - M_1^2$, and write $e_{ij} = \langle \varepsilon_i \varepsilon_j \rangle$.
- This results in the covariance equations $C_{ij} = a_i a_j \left(T + e_{ij}\right)$. These are symmetric in their indices.
- The diagonal equations yield the error variances σ_i^2, the off-diagonal equations the calibration scalings a_i and the common variance T when $e_{ij} = 0$ is assumed.

Collocation model (3)

- In all cases there are n diagonal equations for solving n error variances σ_i^2.
- For triple collocation ($n = 3$) there are 3 off-diagonal covariance equations for determining a_2, a_3, and T – precisely as many equations as unknowns.
- For quadruple collocation ($n = 4$) there are 6 off-diagonal covariance equations to solve for 4 unknowns a_2, a_3, a_4, and T. There are 15 possible combinations of selecting 4 equations. Such a combination will be referred to as a **model** in what follows.
- Of the 15 quadruple collocation models, 12 have a solution and 3 are unsolvable.
- Besides the calibration scalings and common variance, a quadruple collocation analysis also yields 2 additional error covariances e_{ij}.
- We will systematically study all possible solutions without attaching a prior meaning to the two additional error covariances.

Representativeness errors (1)

- Observation systems generally have different spatio-temporal sampling characteristics, resulting in so-called representativeness errors: a system with coarse resolution will miss signal that is detected by systems with finer resolutions. This introduces error correlations (Stoffelen, JGR 1998).
- Suppose that the systems are ordered according to resolution: system 1 the finest resolution, system n the coarsest.
- Let r_i^2 be the representativeness error of system i with respect to system $i+1$, i.e., the extra variance measured by system i because of its finer resolution with respect to system $i+1$.
- Modify the covariance equations to $\bar{C}_{ij} = C_{ij} - \sum_{k=\max(i,j)}^{n-1} r_k^2 = a_i a_j \left(T + e_{ij}\right)$. This handles the representativeness errors.

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Representativeness errors (2)

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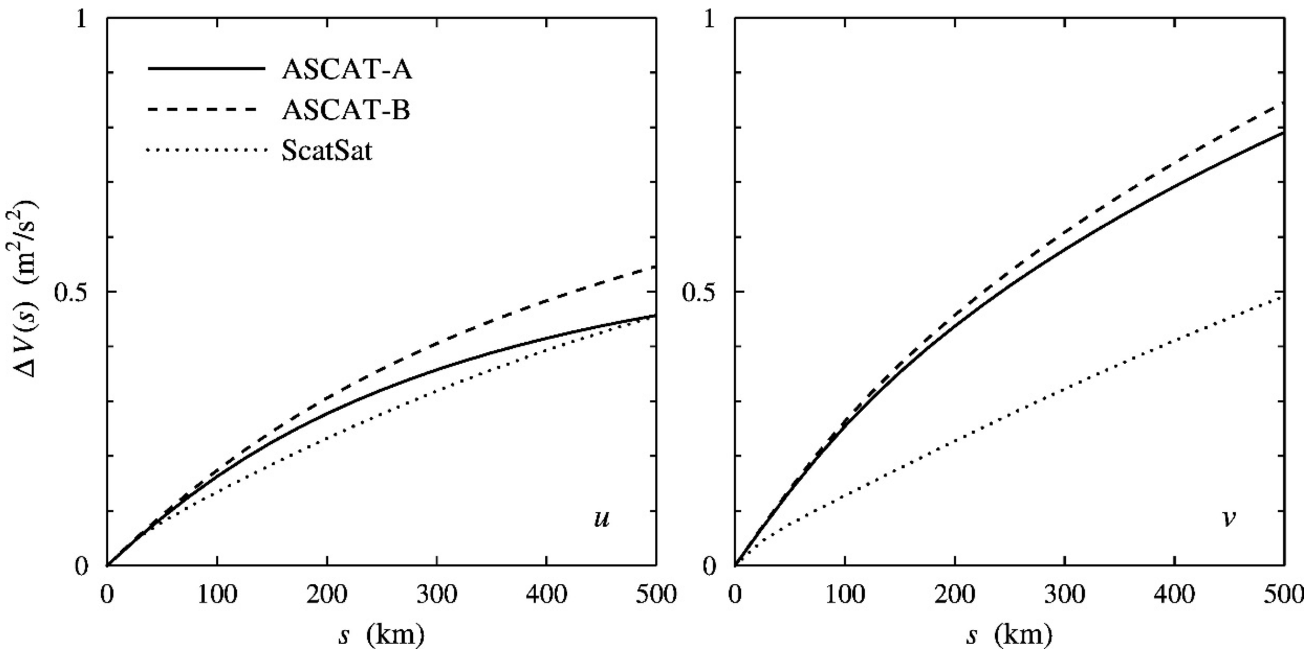

For u (left hand panel), ASCAT-A, ASCAT-B, and ScatSat have almost the same representativeness errors, so their differences in representativeness are probably small. This indicates that their resolution in u is likely almost the same.

For v (right hand panel), ScatSat has a lower representativeness error than the ASCATs, pointing at a poorer resolution of ScatSat caused by 2DVAR ambiguity removal in combination with the Multi Solution Scheme (which is needed to reduce noise).

- Representativeness errors may be obtained from differences in spatial variances, $\Delta V(s)$, at various scales s.
- The figure shows $\Delta V(s)$ of the three scatterometers w.r.t. the ECMWF background. The representativeness error of ASCAT-A w.r.t. ScatSat is the vertical difference between the solid and dotted curves – like an energy level scheme in quantum mechanics.

Results (1a)

- Error variances for buoys, ASCAT-A, ScatSat, and ECMWF (from left to right) as a function of scale s for the wind components u and v.
- Different models may give the same results for a given system, because they form a triple collocation subset.
- For u the buoy and scatterometer results depend little on the representativeness errors (ASCAT and ScatSat representativeness the same), only estimated ECMWF errors are affected. Smallest spread among models at scales around 200 km.
- For v clear effect, with smallest spread between 100km and 200 km.
- One may assume that the smallest spread represents the best fitting error model, hence the best estimate of representativeness error scales

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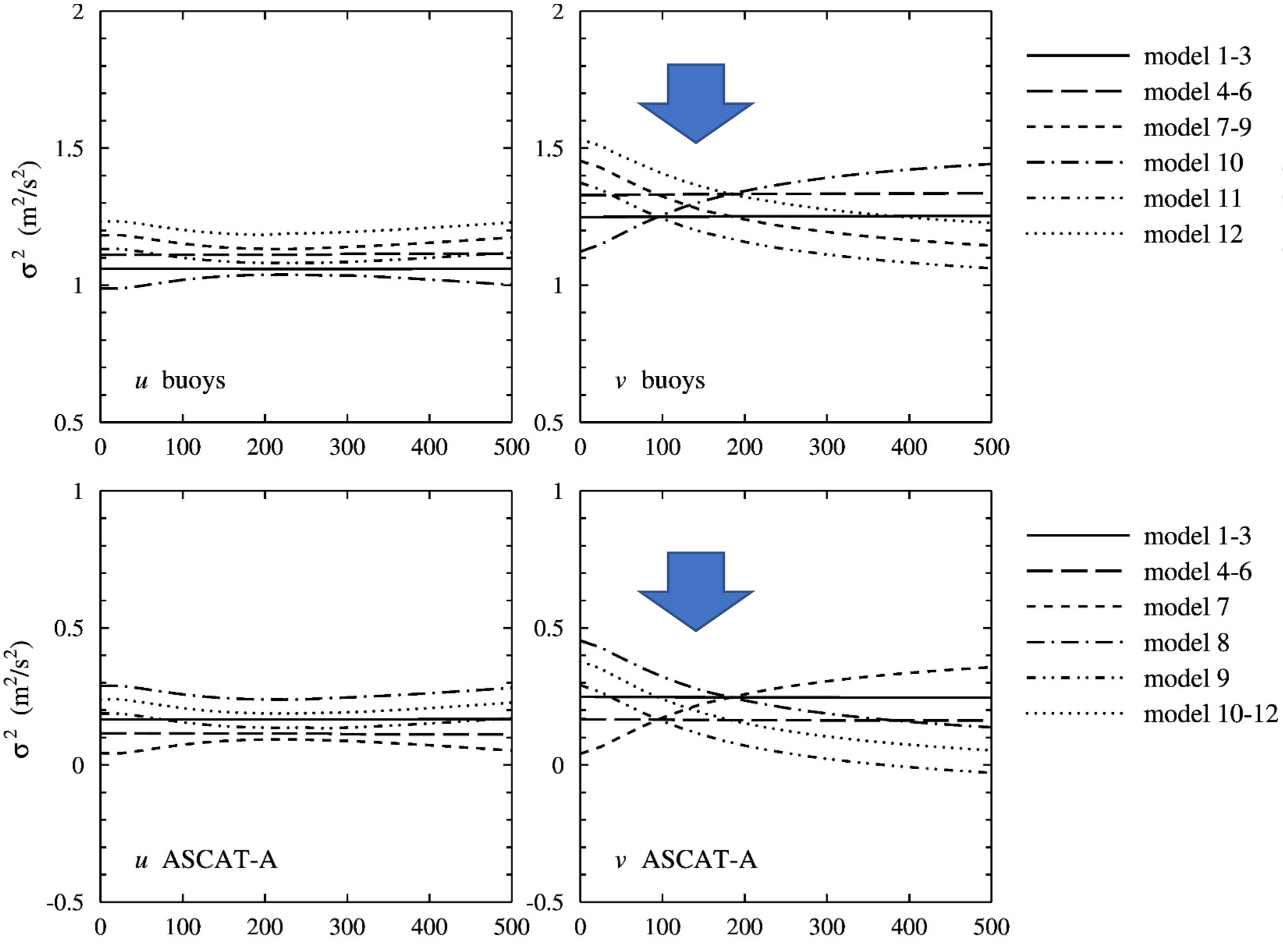
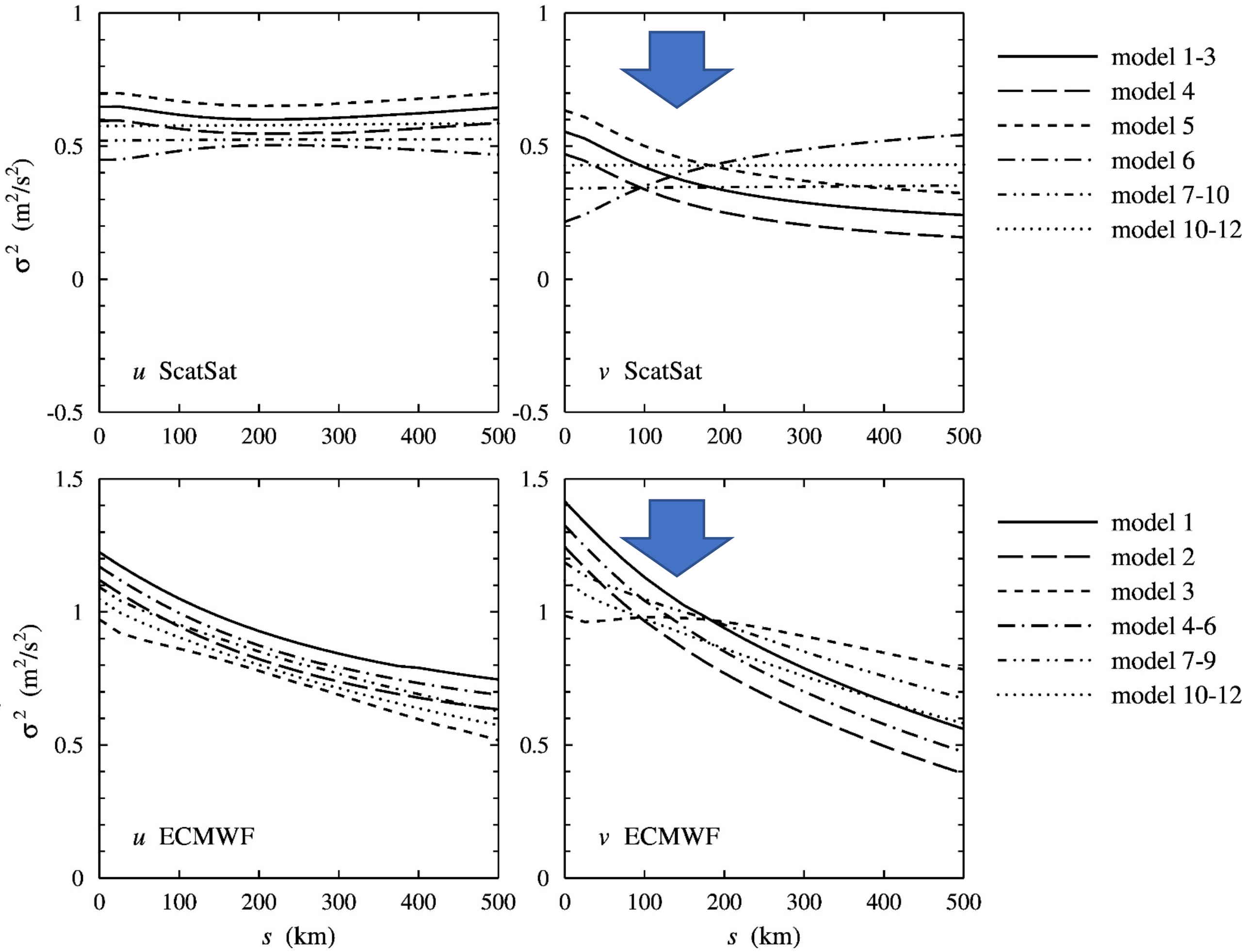
Results (1b)

- For u strongest effect of representativeness error for ECMWF
- Minimum spreading among models for v at scales between 100 km and 200 km





Results (2a)

- Buoy/ASCAT-B/ScatSat/ECMWF results.
- Comparable to the previous slide, except that there is no clear minimum spreading among the models for u, and that the minimum spread for v is at scales around 100 km.

- Note that the ASCAT-B error variance in u becomes unrealistically small for model 7 (dashed curve) at zero scale, i.e., when representativeness is neglected. The same applies for a number of models in v when representativeness is exaggerated!
- Representativeness or small error correlations are important!

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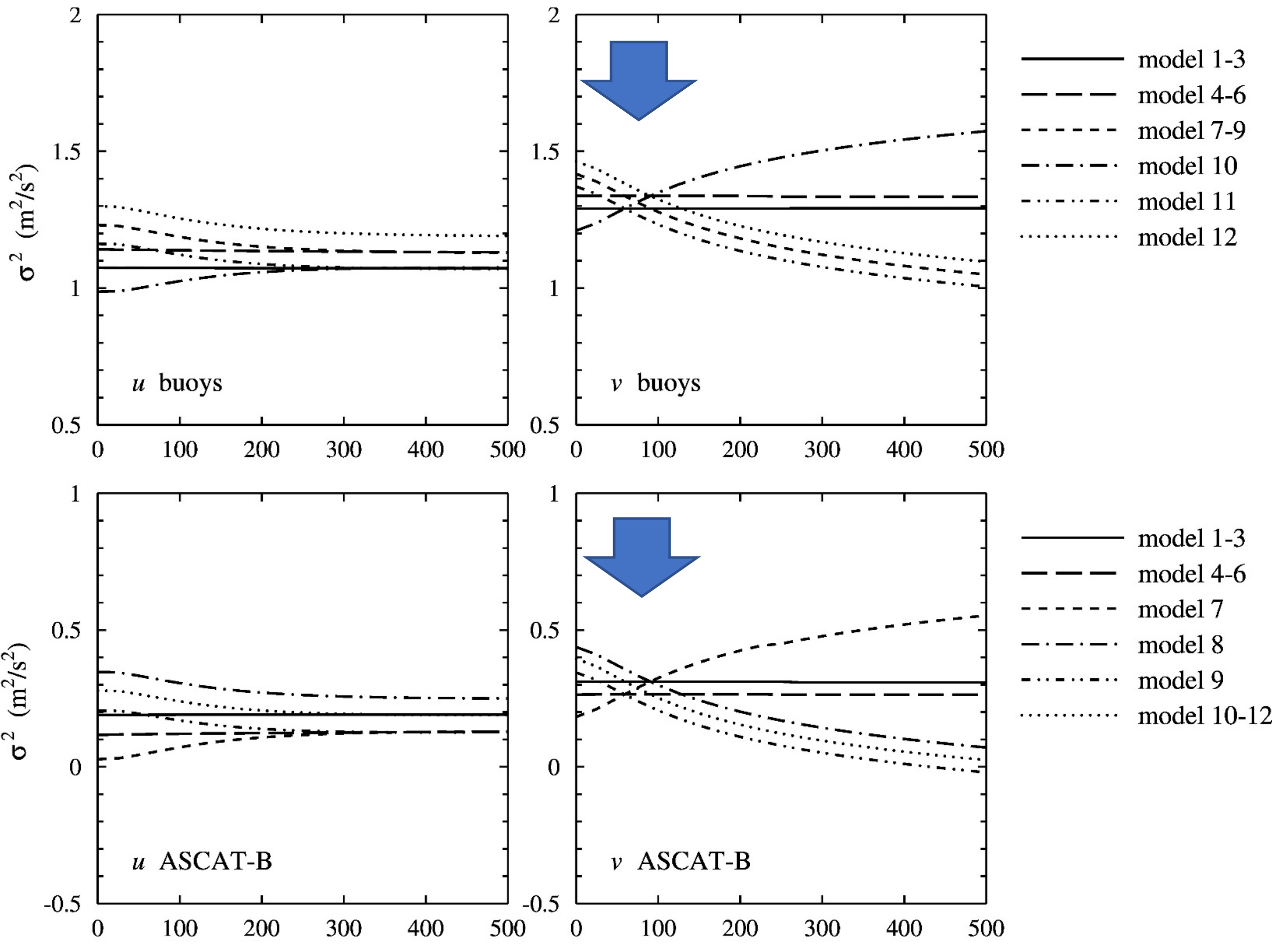
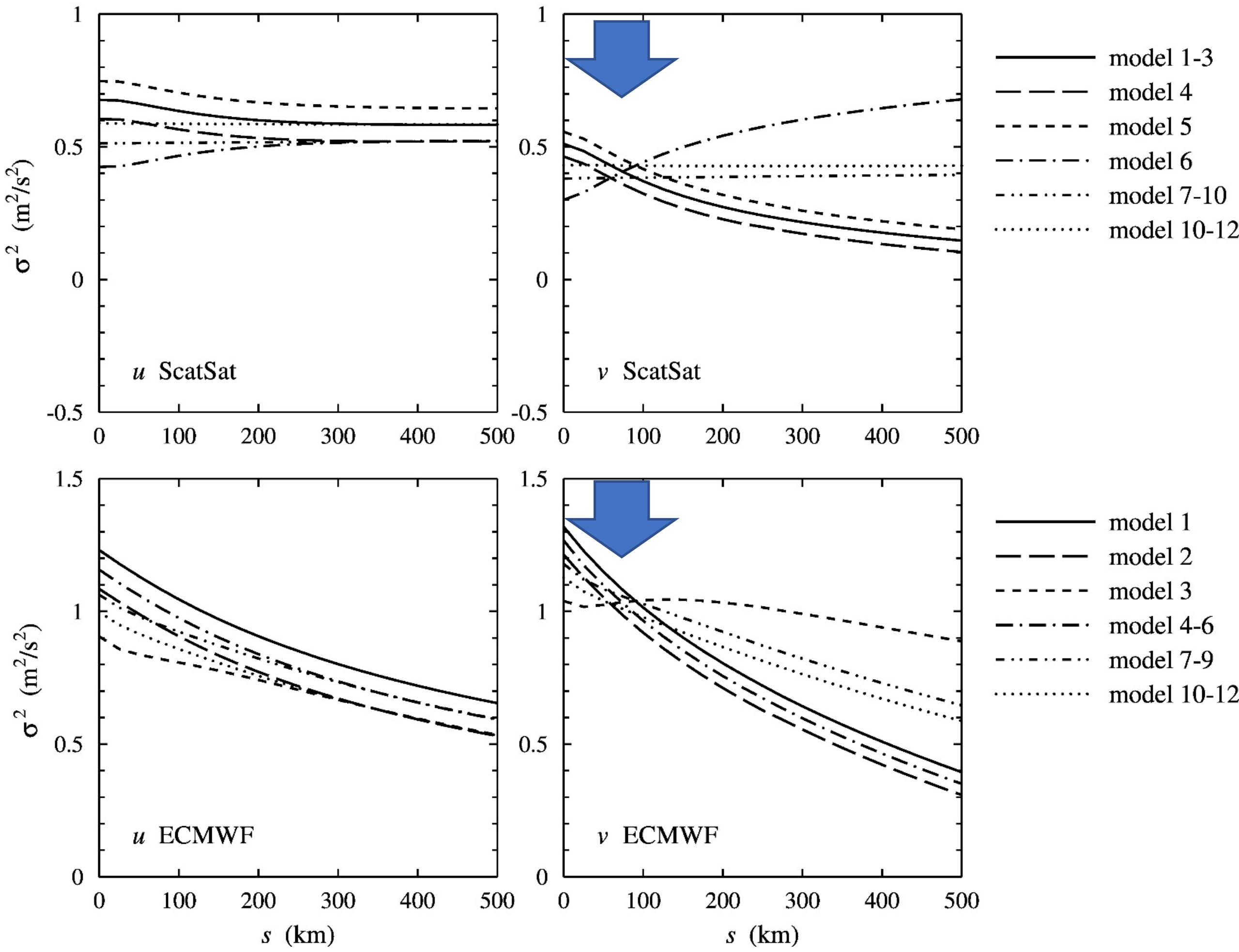
Results (2b)

- For u strongest effect of representativeness error for ECMWF
- Minimum spreading among models for v at scales around 100 km



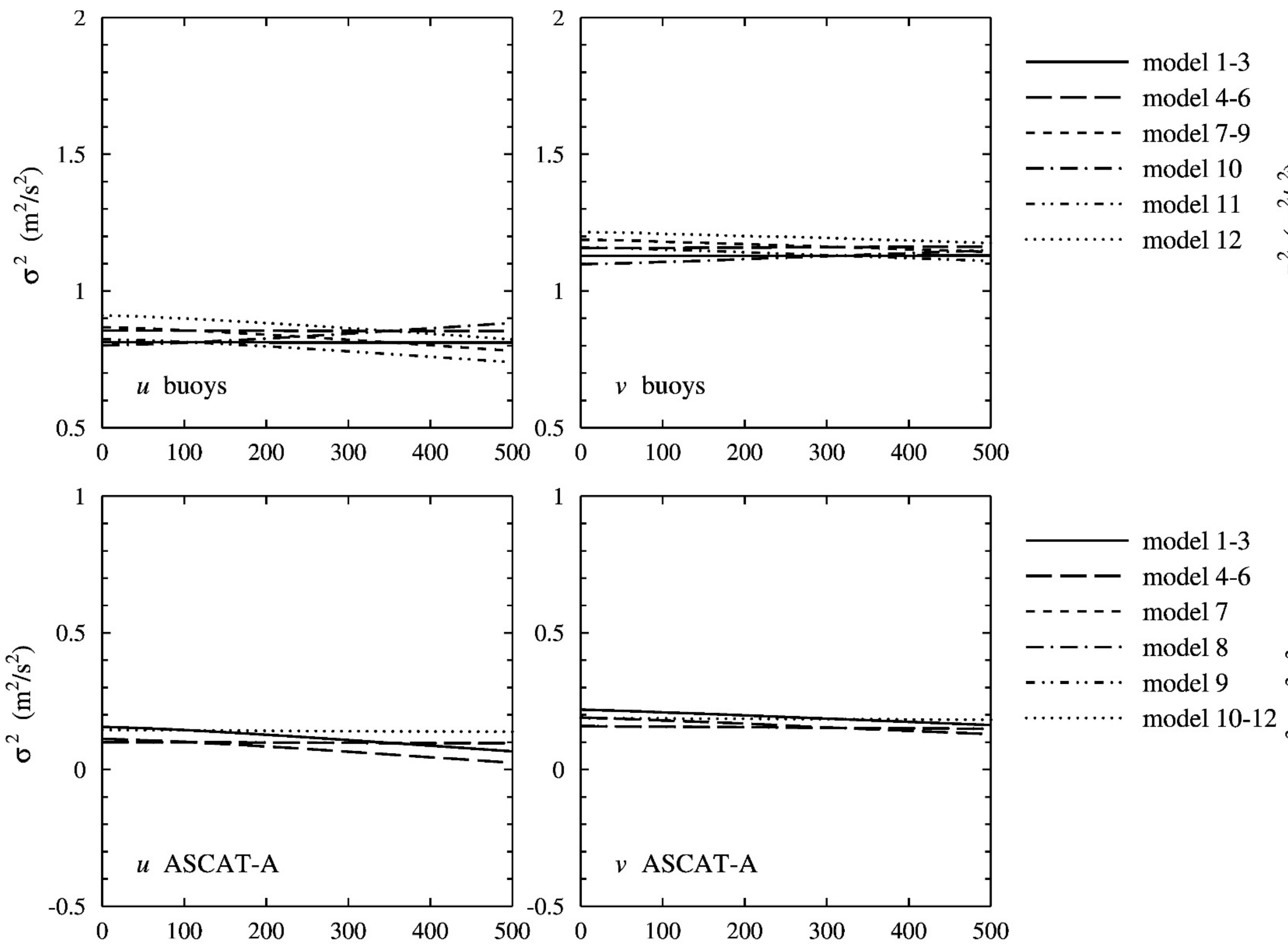


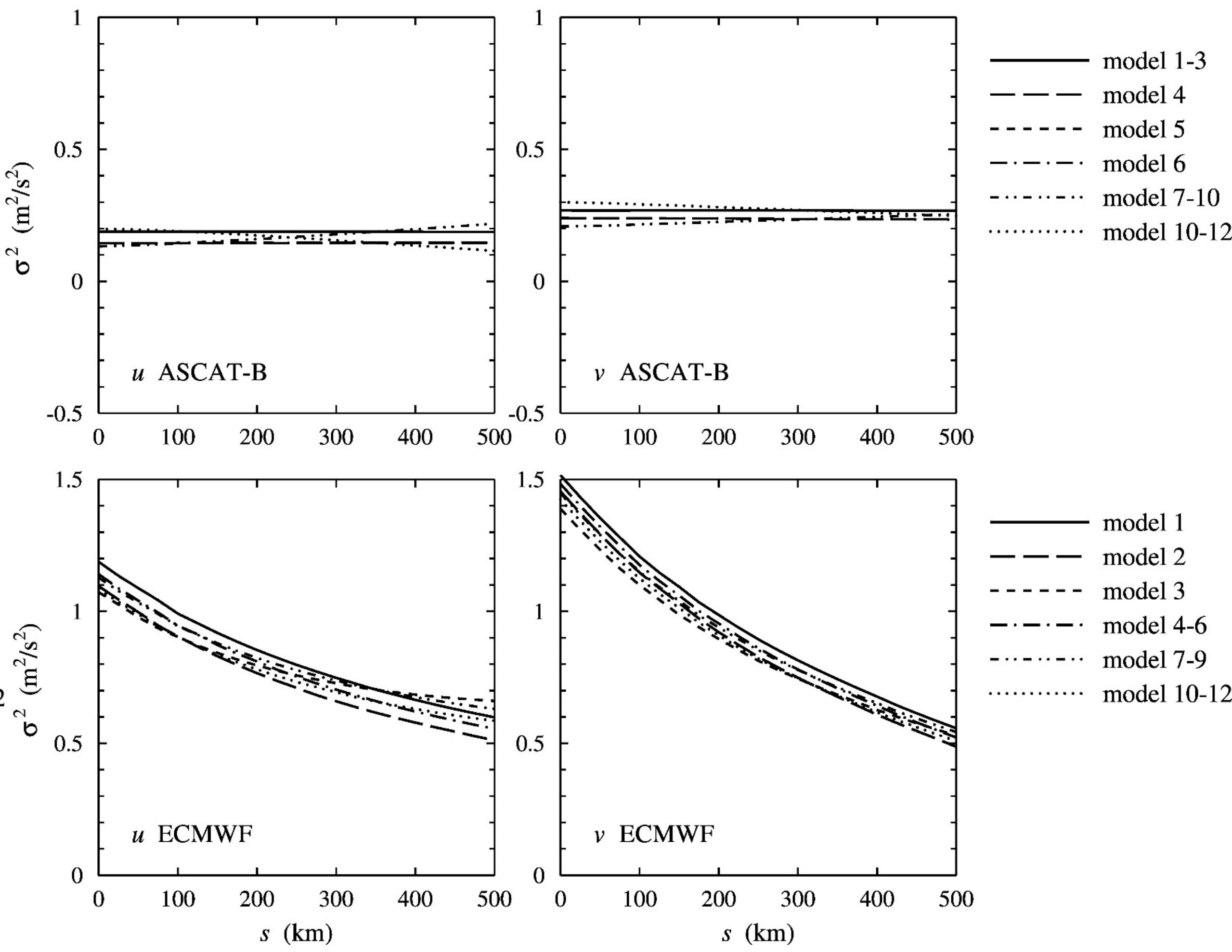
Results (3a)

- Buoys/ASCAT-A/ASCAT-B/ECMWF results.
- Small spread among models.
- Representativeness not important (because it is almost the same for ASCAT-A and ASCAT-B), except for ECMWF.
- Results for error variances agree with those from simulated data (see preprint for the simulations)
- ASCAT-A and ASCAT-B provide independent measurements, but are very consistent

Results (3b)

- Little effect of representativeness error, except for ECMWF





Discussion (1)

- What about the two additional error covariances?

  The additional covariances bear no relation with the representativeness errors, because these appear in more than two error covariances.

- But you only need r_2^2 and r_3^2! Can't you modify the error model?

  Yes, that is possible, but the resulting solution is numerically so unstable that it has no practical use. See Appendix C of the preprint for more information.

- What then is the use of a quadruple collocation analysis?

  It clearly shows the consistency of the underlying error model. The spreading in the results shows that the error model is not perfect. Yet, the spreading is not dramatically large when representativeness is included properly and thus gives a good indication of the reliability of the results.

Triple collocation subresults

Subset	Buoys		ASCAT-A		ScatSat		ECMWF	
	σ_u	σ_v	σ_u	σ_v	σ_u	σ_v	σ_u	σ_v
bAS	1.03	1.12	0.41	0.49	0.78	0.65	--	--
bAE	1.06	1.15	0.34	0.41	--	--	0.94	1.03
bSE	1.09	1.21	--	--	0.72	0.59	0.92	1.03
ASE	--	--	0.43	0.49	0.76	0.65	0.90	0.98
range	0.06	0.09	0.09	0.08	0.06	0.06	0.04	0.05

b: buoys
A: ASCAT-A
S: ScatSat
E: ECMWF

- Error variances (in m^2/s^2) from triple collocation
- Representativeness errors from spatial variances at 200 km for u and 100 km for v
- The range (bottom row) gives the range in the values of the error variances

- Results consistent within 0.05 m^2/s^2
- Same consistency for buoys/ASCAT-B/ScatSat/ECMWF and buoys/ASCAT-A/ASCAT-B/ECMWF

Conclusions

- The quadruple collocation problem can be solved in 12 ways
- The additional two error variances are of little use, except when one knows in advance which error correlations can safely be neglected
- Representativeness errors are important, but must be estimated using a different method
- Differences between the 12 solutions indicate imperfections in the error model; however, in this study the error model is consistent within 0.05 m^2/s^2 for the error variances of the instruments