How QNSE theory and scatterometry can improve various data products

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Turbulence is all around us

- Campaigns on dispersion in the atmosphere and ocean reveal Richardson diffusion regime – strong evidence of Kolmogorov turbulence
- May feature either direct or inverse cascade
- If we know the rate of the cascade, ε, we can compute eddy viscosity and eddy diffusivity (mixing coefficients) even in anisotropic turbulence
- Estimating mixing coefficients is one of the most difficult problems for NWP, modeling of geophysical and planetary circulations
- Mixing coefficients should recognize the interaction between turbulence and waves
- Three main types of waves: internal (N), inertial (f), Rossby (β)
- What are controlling scales that involve turbulence (ε) and waves?

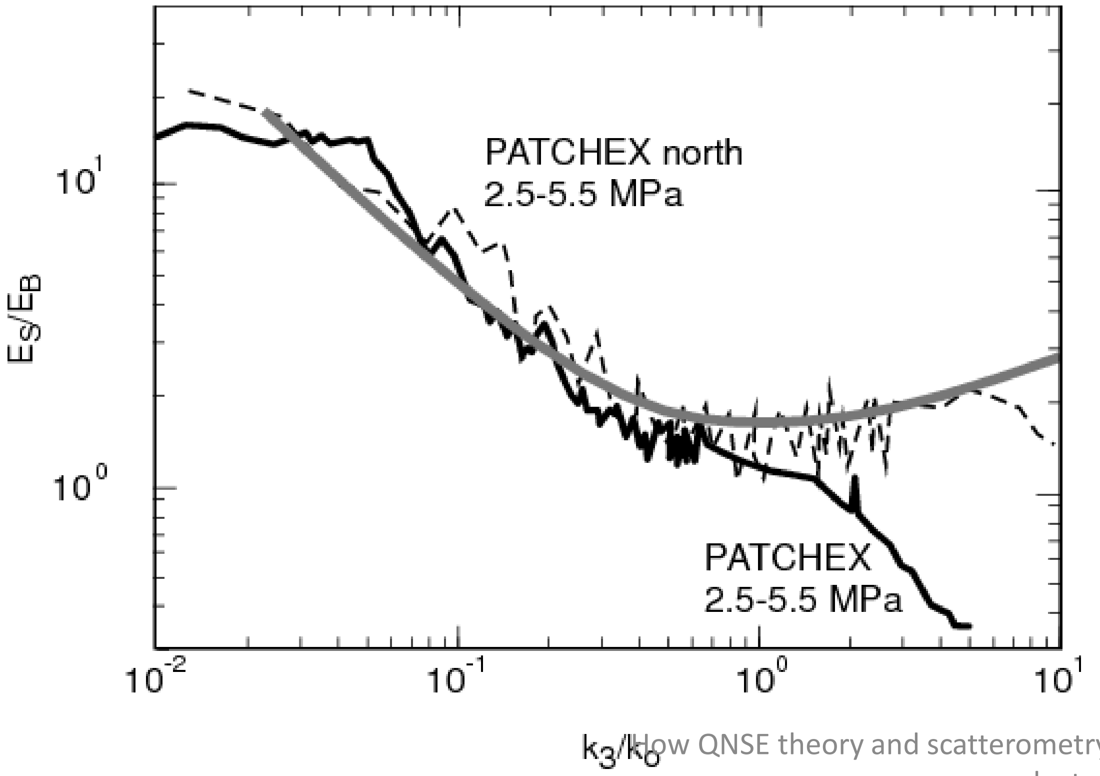
Controlling (crossover) scales

- Ozmidov $L_O = (\varepsilon/N^3)^{1/2}$
- Woods $L_\Omega = (\varepsilon/f^3)^{1/2}$
- Transitional $L_\beta = (\varepsilon/\beta^3)^{1/5}$
- Advantageous over $|\zeta|/f$ which does not delineate length scales
- On smaller scales, turbulence prevails $\Rightarrow$ Kolmogorov spectra
- On larger scales, wave-related parameters dominate, spectra become universal
- Can be used as performance outliers

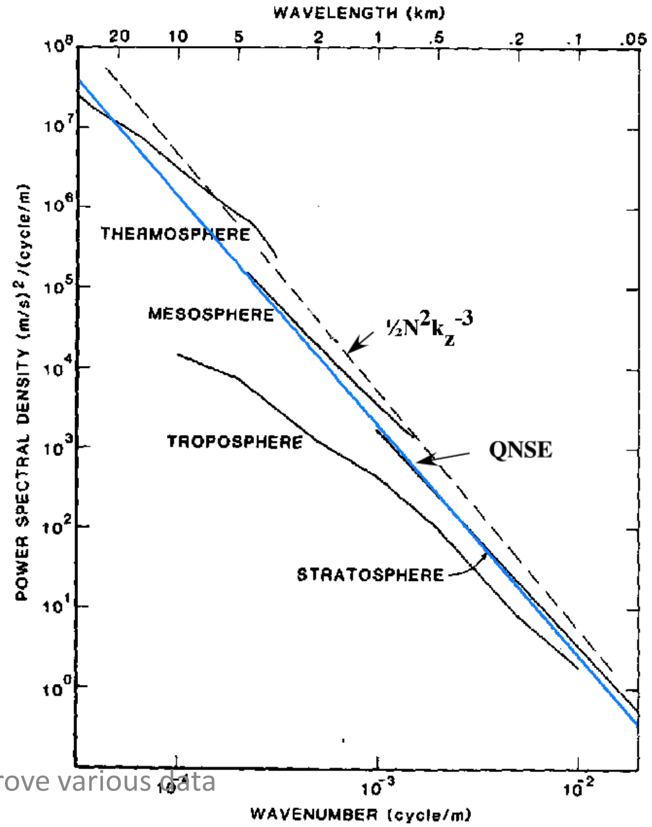
Universal vertical spectrum of the horizontal velocity

$$
\begin{aligned}
E_1(k_3) &= 0.626\,\epsilon^{2/3}\,k_3^{-5/3} + 0.214\,N^2 k_3^{-3} \\
&= 0.626\,\epsilon^{2/3}\,k_3^{-5/3}\left[1 + 0.34(k_3/k_O)^{-4/3}\right]
\end{aligned}
$$

Gregg, Winkel, Sanford, JPO, 1993

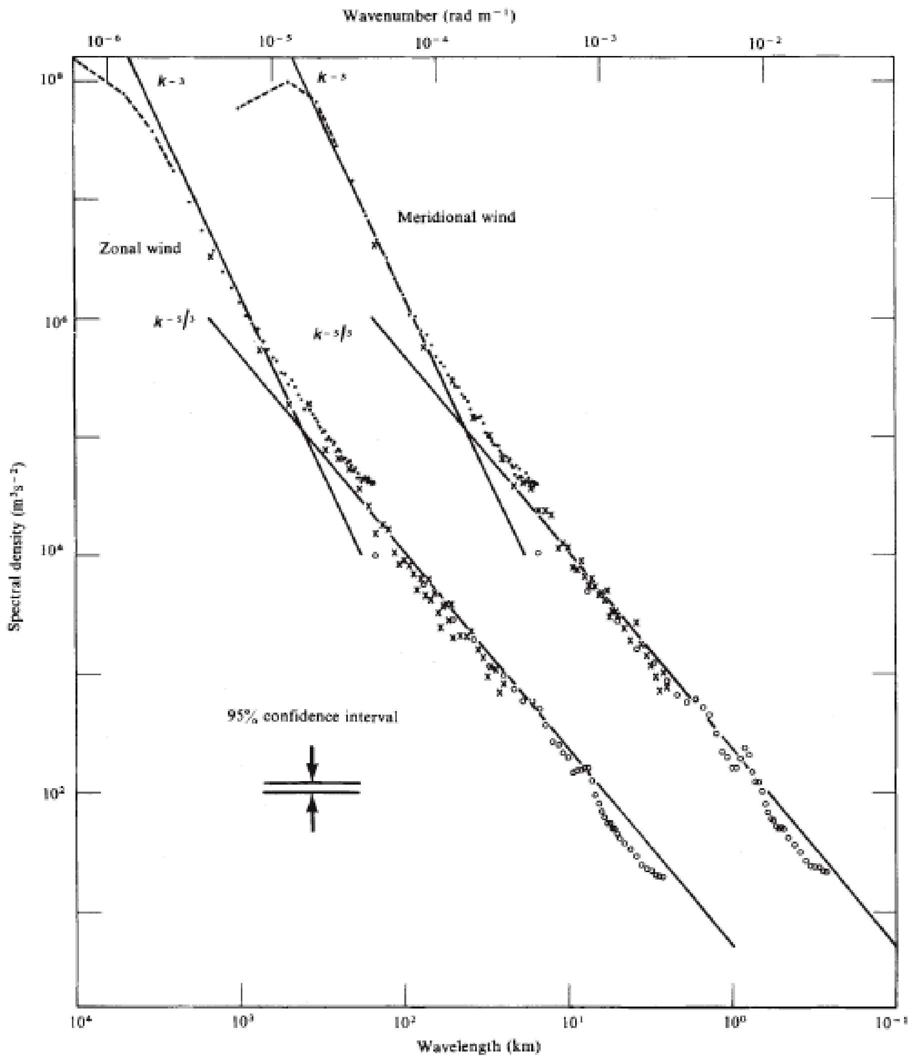


Smith, Fritts, Van Zandt, JAS, 1987

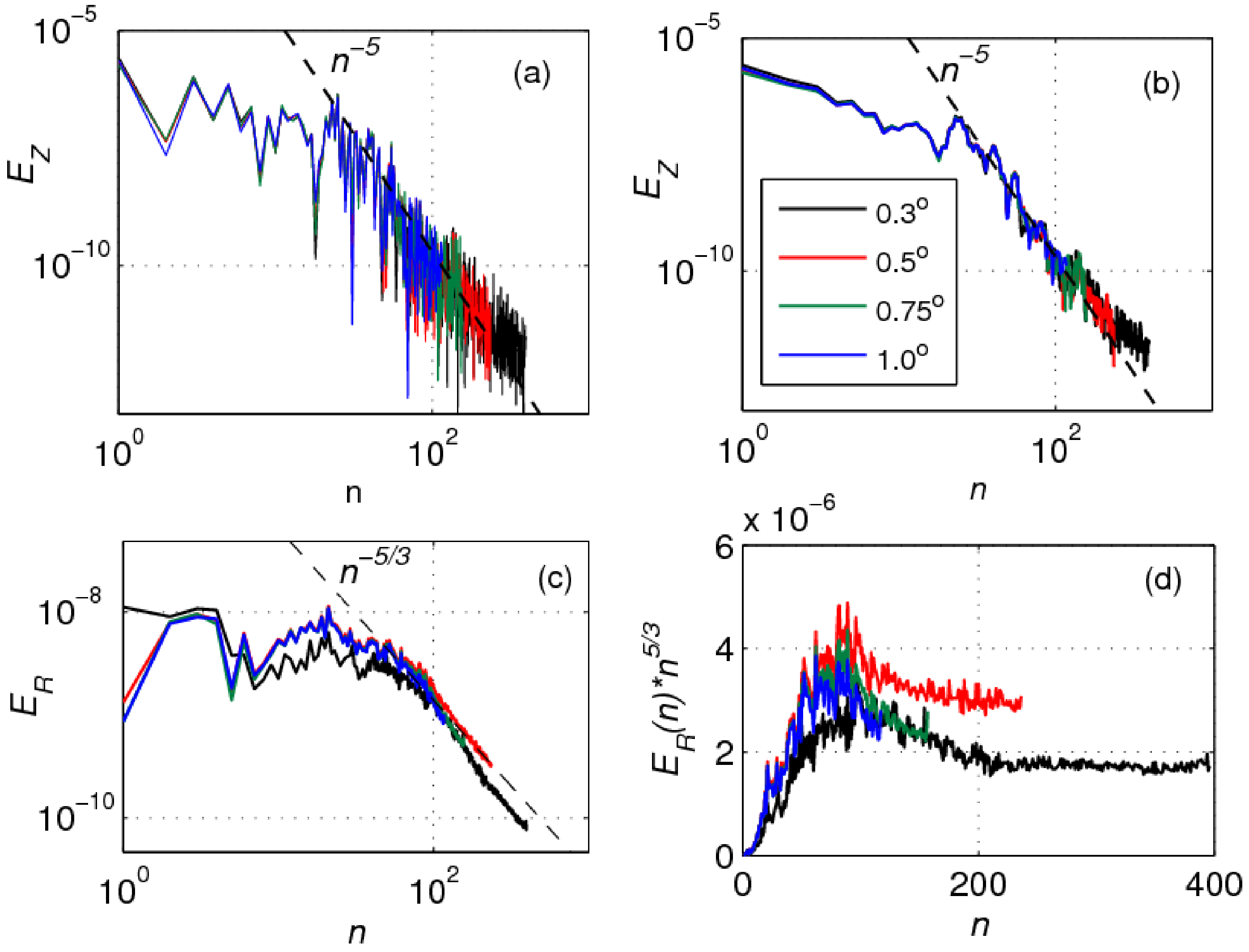


Universal horizontal spectrum of the horizontal velocity

- Upper troposphere and lower stratosphere – Global Atmospheric Sampling Program (GASP; Nastrom & Gage, 1985), Measurement of Ozone by Airbus In-Service aircraft (MOSAIC; Marenco, 1998, Lindborg, 1999)
- **Nastrom & Gage spectra, or canonical spectra**
- On synoptic scales (2-3 x 10^3 to 500km), the slope is k_h^{-3}, k_h – the horizontal wavenumber
- On mesoscales (500 to 10 km), spectrum transitions to the Kolmogorov $k_h^{-5/3}$ slope
- Spectra are remarkably universal throughout the troposphere and stratosphere, the seasons, but are dependent on latitude
- Spectral amplitude decreases towards the equator

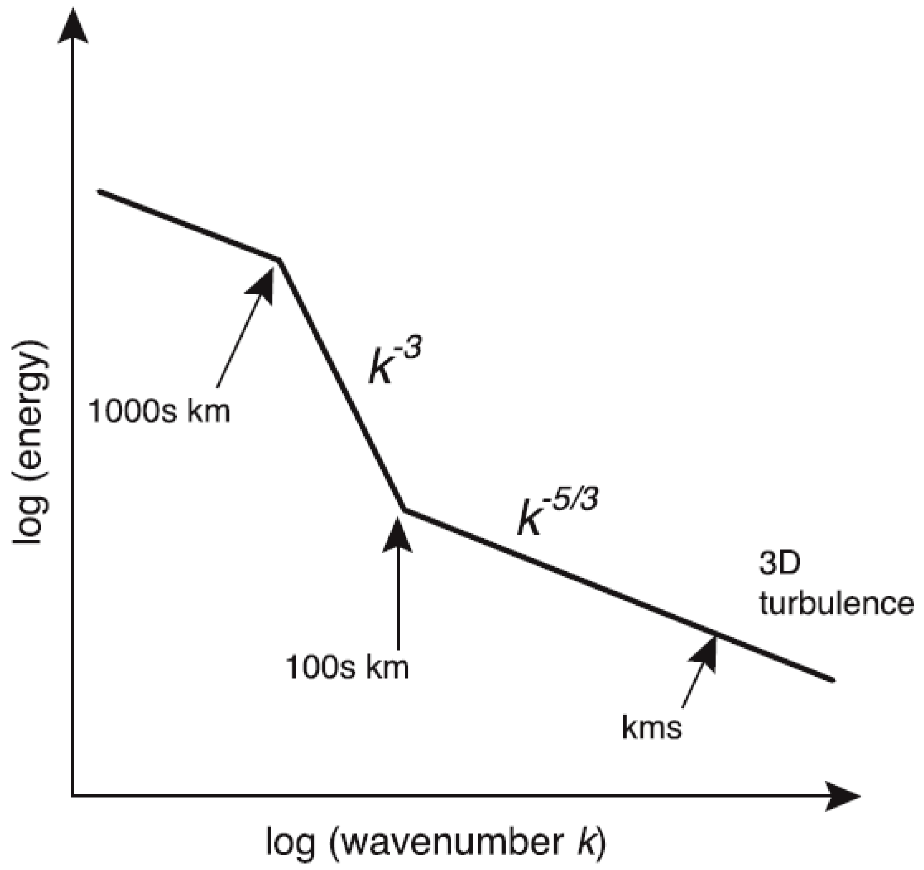


Zonal and residual spectra on Jupiter from Cassini data



The physics of the Nastrom & Gage spectrum

- Still no consensus
- Most common – k_h^{-3} branch is due to direct enstrophy cascade, following Kraichnan's theory of 2D turbulence and Charney's (1971) scaling
- But – O'Gorman & Schneider (2007) showed that enstrophy cascade is unnecessary
- Cho & Lindborg (2001) – there is no explanation of the k_h^{-3} range other than Charney's two-dimensionalization
- Lovejoy (2009) – k_h^{-3} range is **not** present; is due to analysis errors
- Dynamics of $k_h^{-5/3}$ range is even less clear
- Was considered to be due to inverse cascade – but Lindborg (1999) used structure functions to demonstrate direct cascade
- Lindborg suggested stratified turbulence – but Skamarock et al. (2014) showed it unsupported
- Cho & Lindborg (2001) – the Coriolis force may be crucial but nearly impossible to account for via the energy equation



With no clear understanding of the physics, the N&G spectrum has been used as an outlier

- The dynamics has implications for forecast error growth, spinup time scales, filtering and subfilterscale physics (e.g., Skamarock 2004; Shutts 2005)
- Atmospheric models' ability to reproduce N&G spectrum serves as validation of the correctness of model's formulation, implementation, and configuration
- KE spectrum provides a measure of model's effective resolution and filter effects
- Lovejoy et al. (2009) brought up the importance of turbulence and its *anisotropization* in interpretation of the N&G spectra
- Suggested that in anisotropic turbulence, structures progressively flatten out with increasing scale and may obey a power law that obliterates the need in invoking the 3D-2D transition
- ***'the entire mainstream view of the atmosphere has fundamentally been coloured by the assumption of isotropic turbulence'***

Yano (2010):

"…My naive feeling is that an elaborated use of a ***renormalization group (RNG)*** theory might potentially lead to a necessary theoretical breakthrough, but I should not be too speculative."

He was not too speculative…

RNG-based Quasi-Normal Scale Elimination (QNSE) theory indeed offers a breakthrough

- ❑ Yields analytical expression for the N&G spectrum and explains its major features, both qualitatively and quantitatively
- ❑ Considers 3D fluid occupying infinite domain; dynamics is represented by full Navier-Stokes and continuity equations in a coordinate frame rotating with the angular velocity $\mathbf{\Omega}$
- ❑ QNSE is an algorithm of successive coarsening of the flow domain by cyclically eliminating small shells of wave number modes and computing compensating corrections to the viscosity
- ❑ **No quasi-geostrophy is implied**

Mathematical formulation

$$\frac{\partial v_\alpha}{\partial t} + (\mathbf{v}\cdot\nabla)v_\alpha + (\mathbf{f}\times\mathbf{v})_\alpha = \nu_0\nabla^2 v_\alpha - \frac{\partial P}{\partial x_\alpha} + \xi_\alpha^0$$

$$\frac{\partial v_\alpha}{\partial x_\alpha} = 0$$

Space-time Fourier transform (d=3 is the dimension of space) :

$$v_\alpha(\mathbf{x},t) = \frac{1}{(2\pi)^{d+1}} \int_{k\le k_d} \mathrm{d}\mathbf{k} \int \mathrm{d}\omega\, v_\alpha(\omega,\mathbf{k}) \exp[i(\mathbf{kx}-\omega t)]$$

Solution's steps

The crossover between turbulence and inertial waves is on scales $O(L_\Omega)$

$L_\Omega = (\varepsilon/f^3)^{1/2}$ is the Woods scale, $f = 2\Omega$ is the Coriolis parameter

Rotation leads to the development of the inverse cascade on scales > L_Ω

The procedure of the coarse-graining:

- Introduce the dynamic dissipation cutoff wavenumber, Λ, a small shell $\Delta\,\Lambda$, $\Delta\Lambda/\Lambda \ll 1$, `slow' and `fast' modes
- Compute $O(\Delta\Lambda)$ correction to the inverse Green function by ensemble averaging of the fast modes over $\Delta\Lambda$. This correction generates $O(\Delta\Lambda)$ accruals to all renormalized viscosities while preserving the analytical form of the governing equations

- All viscosities are updated and the process moves forward towards elimination of the next shell $\Delta\Lambda$

The analytical solution

$$\frac{\nu_h}{\nu_n} = 1 - \frac{41}{252} Ro(k)^{-2} = 1 - 0.77(k / k_\Omega)^{-4/3}$$

$$\frac{\nu_z}{\nu_n} = 1 - \frac{73}{1260} Ro(k)^{-2} = 1 - 0.27(k / k_\Omega)^{-4/3}$$

$$\frac{\nu_3}{\nu_n} = 1 - \frac{37}{1260} Ro(k)^{-2} = 1 - 0.14(k / k_\Omega)^{-4/3}$$

$$\frac{\nu_{3z}}{\nu_n} = 1 + \frac{19}{252} Ro(k)^{-2} = 1 + 0.36(k / k_\Omega)^{-4/3}$$

$$\frac{\kappa_h}{\nu_n} = \alpha + 0.049\, Ro(k)^{-2} = \alpha + 0.23(k / k_\Omega)^{-4/3}$$

$$\frac{\kappa_z}{\nu_n} = \alpha - 0.001\, Ro(k)^{-2} = \alpha - 0.0049(k / k_\Omega)^{-4/3}$$

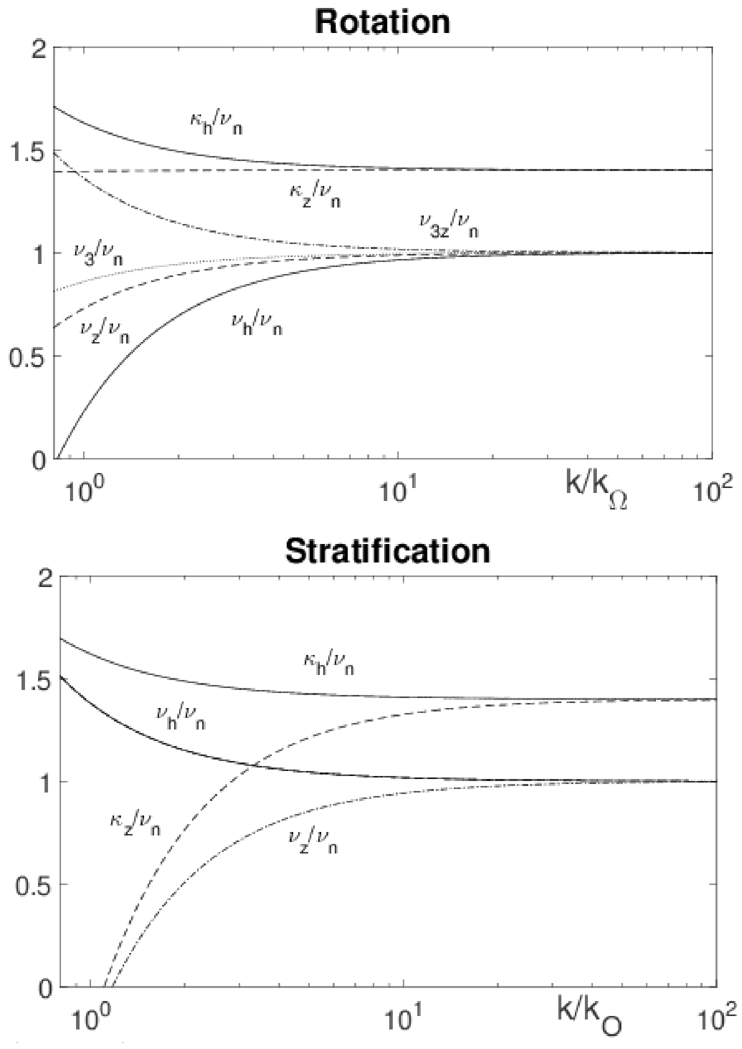
$\alpha = \kappa_n / \nu_n = Pr_{t0}^{-1} \simeq 1.39$ the inverse turbulent Prandtl number in non-rotating flows

Viscosity and diffusivity renormalization

- In turbulence on f-plane introduce the Woods scale

$$L_\Omega = (\varepsilon/f^3)^{1/2}$$

- Analogous to Ozmidov scale



- Weak rotation, $k/k_\Omega = O(1)$
- On small scales, turbulence is isotropic and Kolmogorov-like
- Viscosity undergoes anisotropization and componentality
- 4 renormalized viscosities that act in different directions and on different velocity components
- Horizontal viscosity $\rightarrow 0 \Rightarrow$ indication of the inverse cascade

Nastrom & Gage Spectra

- Zonal spectrum of zonal velocity (longitudinal):

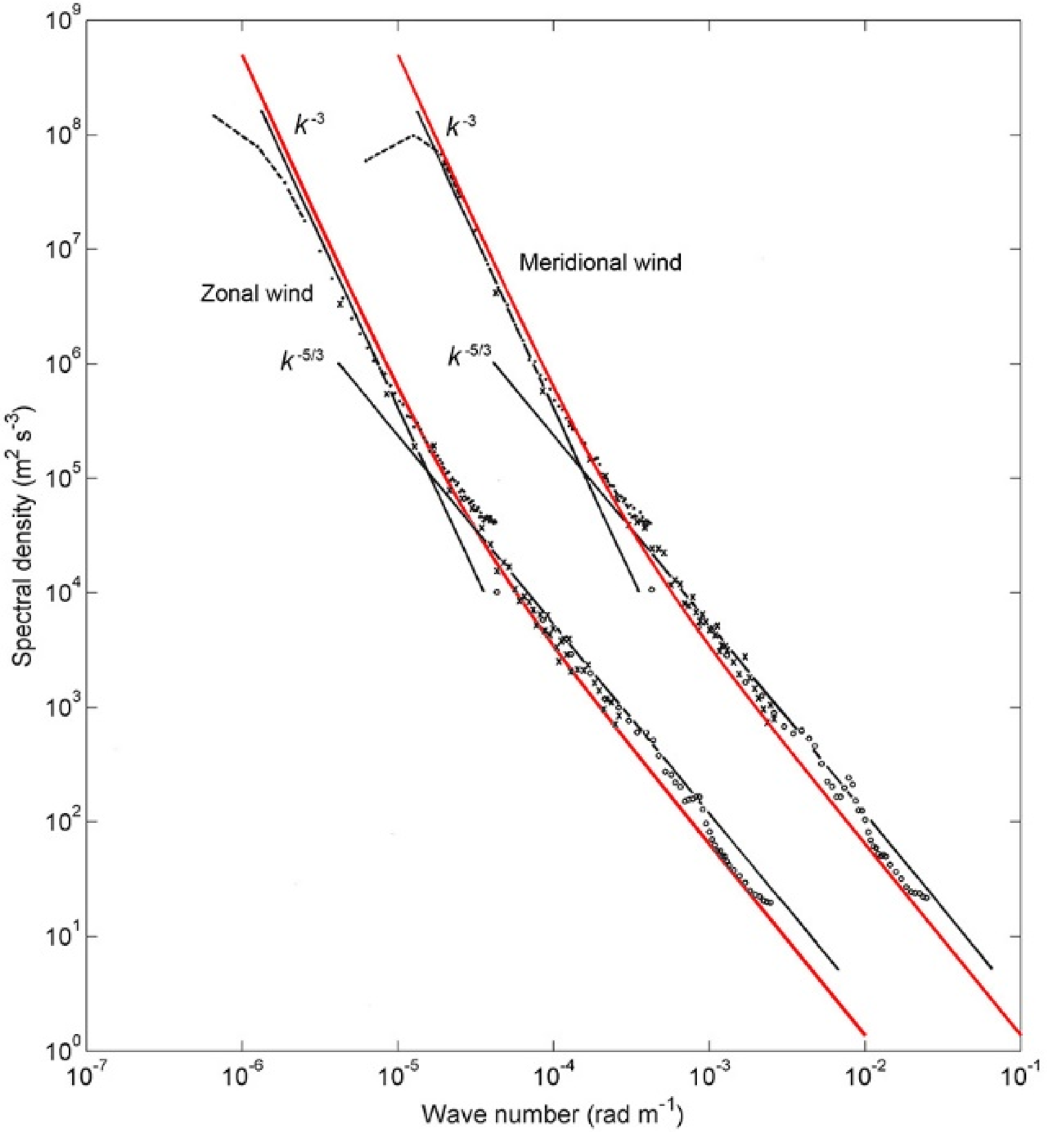
$$E_1(k_1) = 0.47\epsilon^{2/3}k_1^{-5/3} + 0.0926 f^2 k_1^{-3}$$

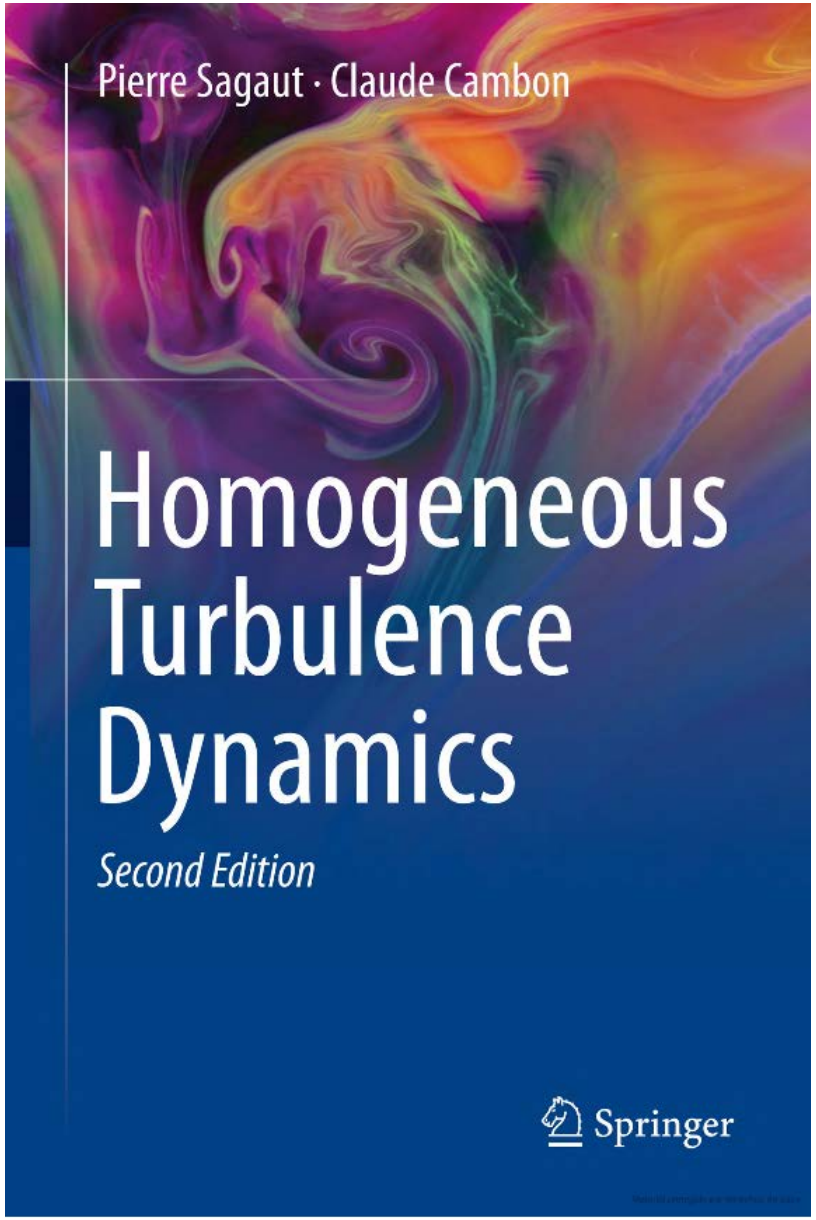
- The dissipation rate $\varepsilon = 5\text{x}10^{-5}\ m^2s^{-3}$

  (after Frehlich & Sharman, 2010)
- Latitude 30°N
- Transverse spectrum:

$$E_2(k_1) = 0.626\epsilon^{2/3}k_1^{-5/3} + 0.240 f^2 k_1^{-3}$$

- The flow is forced on large scales and features direct cascade throughout to small scales
- The physics: Anisotropic turbulence with dispersive (inertial) waves
- Spectra are universal as they depends on f only





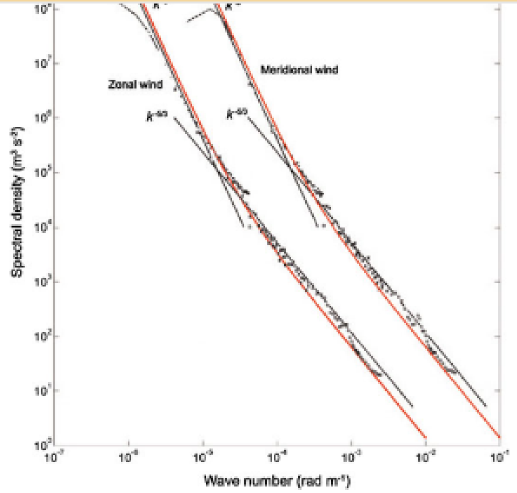


Fig. 7.21 Comparison of the canonical spectra of Nastrom and Gage (1985) with QNSE Eq. (7.58) (red lines). The "canonical" meridional spectrum is shifted one decade to the right. Courtesy of Boris Galperin and Sukoriansky (2017)

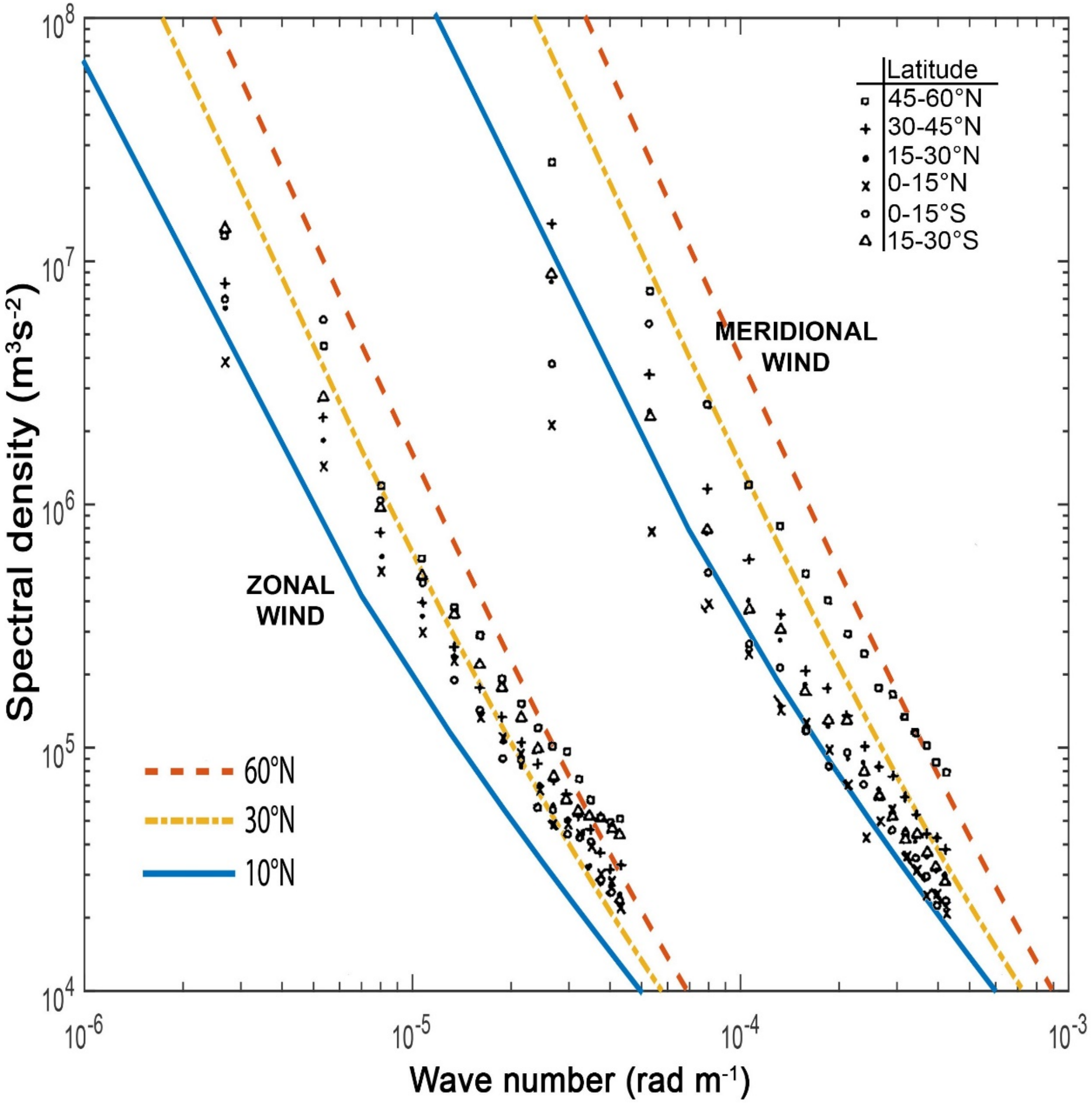
hypothesis of "non-linear formation of structures", supported by the anisotropic statistical approach of Cambon's team, but relevant criticisms by Peter Davidson must be accounted for. The appearance of these structures depends on the range of Rossby and Reynolds numbers, and also on the resolution and effective confinement of the numerical simulation.

A more realistic confinement is present in the DNS by Godeferd and Lollini (1999) on a plane channel rotating about the vertical direction. In addition to a realistic numerical approach to vertical confinement (pseudo-spectral Fourier-Fourier-Chebishev code with no-slip boundary conditions), another motivation was to reproduce the main results of the experiment by Hopfinger et al. (1982), briefly introduced in Sect. 7.1.1. Identification of vortices is illustrated in Fig. 7.22 (top) using both horizontal sections of iso-surfaces (noisy spots in the bottom plane of the figure) and isovalue surfaces of a normalized angular momentum, which is defined in the caption. The latter criterion (Normalized Angular Momentum) was suggested by experimentalists (Marc Michard, Lyon) in PIV for obtaining smooth isovalues. Asymmetry in terms of cyclones-anticyclones is mainly induced by the Ekman pumping near the

How QNSE theory and scatterometry can improve various data products

Dependence of the spectrum on latitude

- Has not been explained
- QNSE provides quantitative explanation
- In good agreement with data
- Suggests that the k_h^{-3} branch may completely disappear near the equator
- Simulations with f=0 (e.g., Durran et al., MWR, 2017; Weyn & Durran, JAS, 2017; Sun et al., JAS, 2017) yield $k_h^{-5/3}$ spectra only
- Directly observed in the data for the ocean near-surface winds (Xu et al., 2011)



Spectra of the ocean near-surface winds

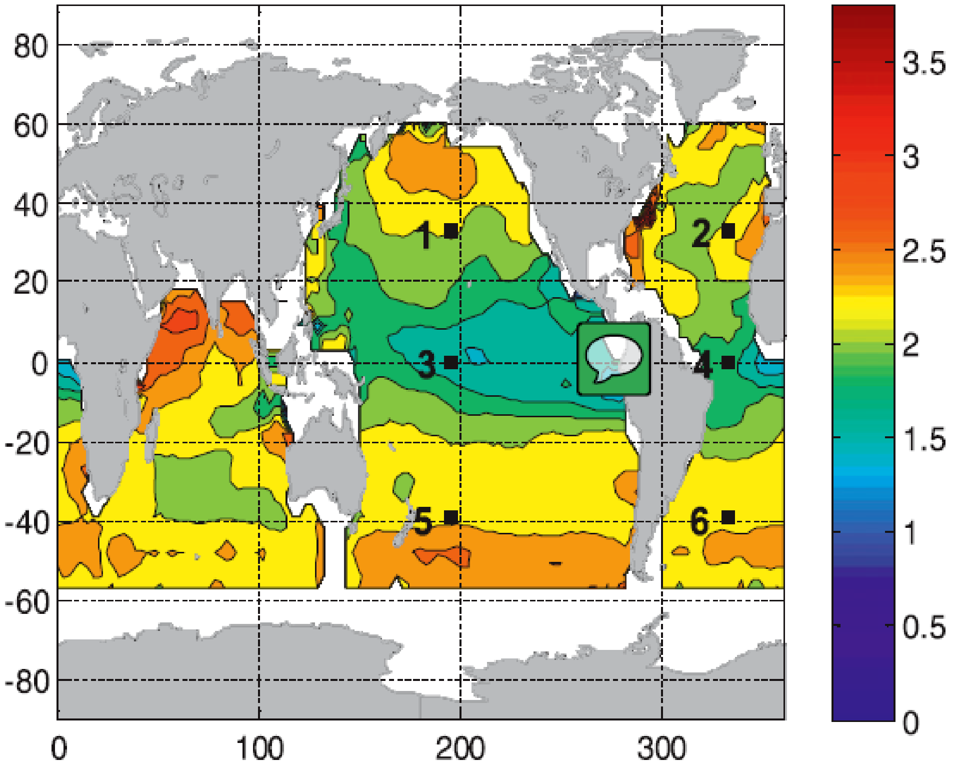
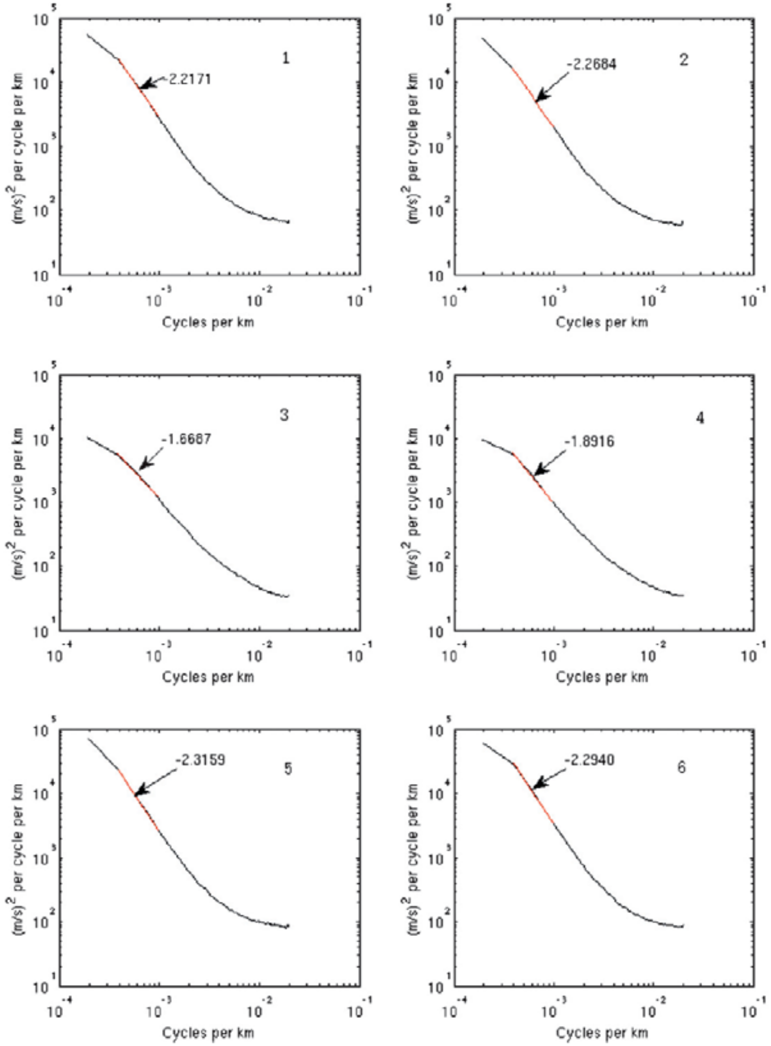
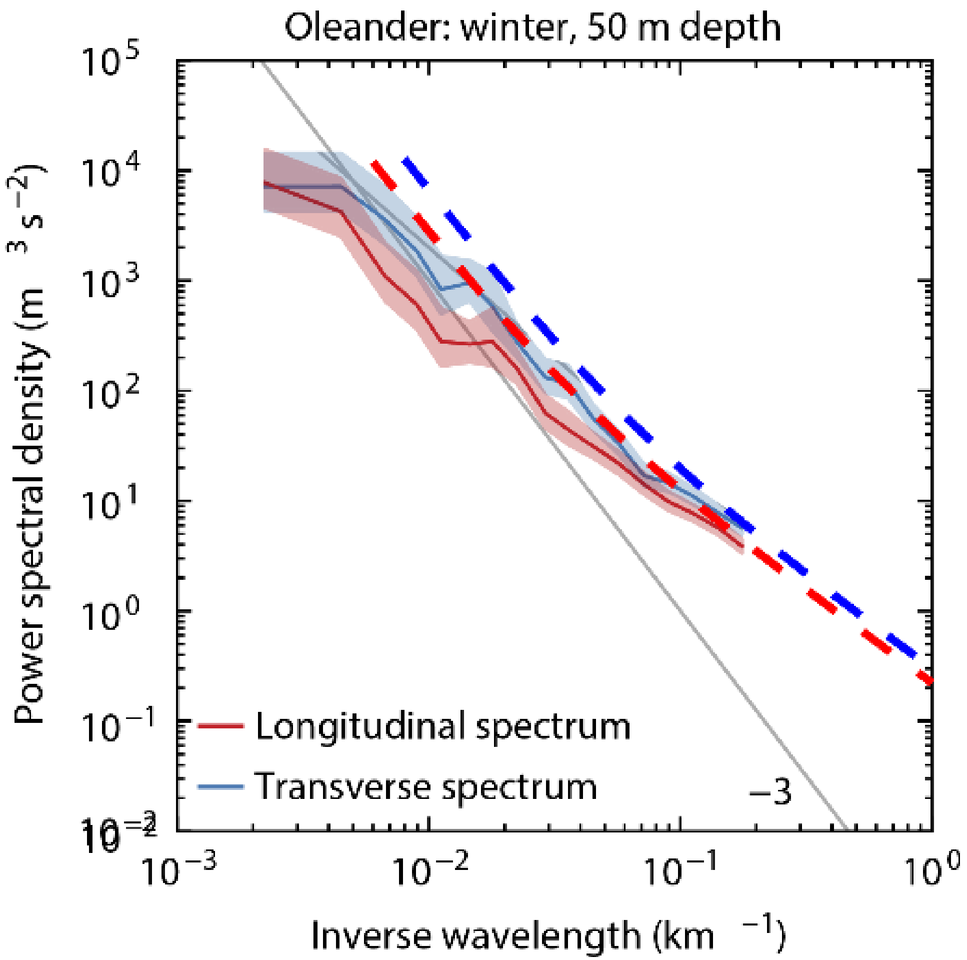


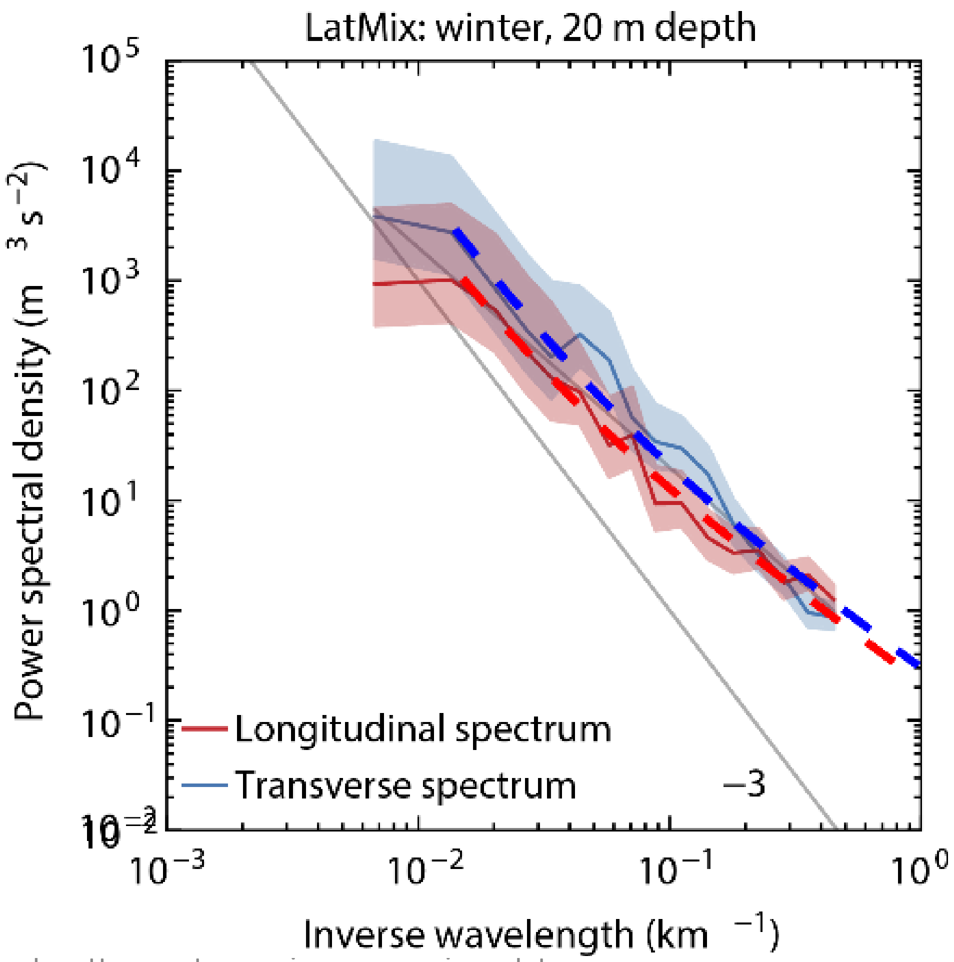
FIG. 2. The global distribution of the spectral slopes of kinetic wavenumber spectrum in the wavelength band of 1000–3000 km estimated from the QuikSCAT scatterometer measurements. The sign of the slopes was reversed to make the value positive.



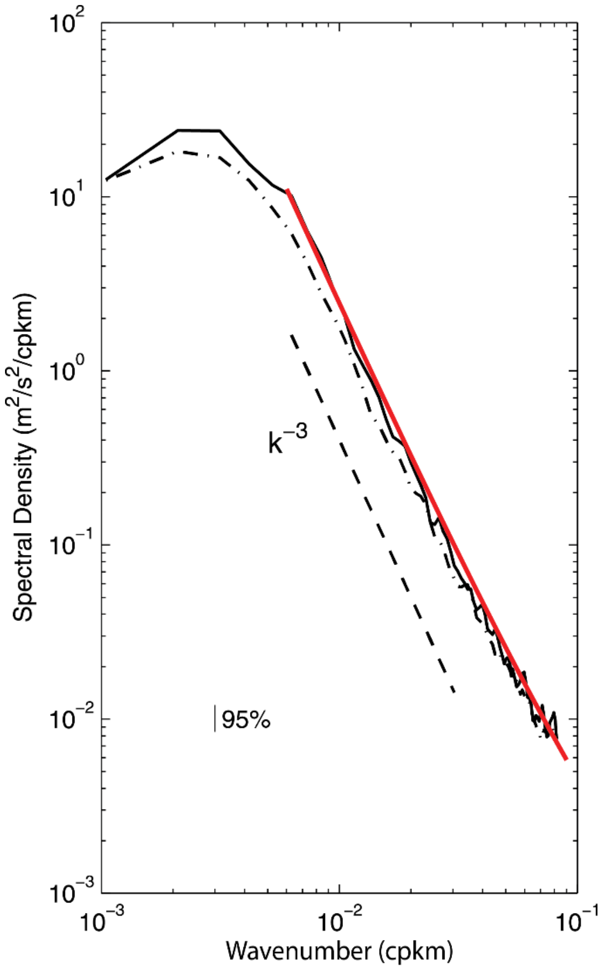
Ocean campaigns

- If the large-scale spectrum depends on f only, one expects to find it in other environments, for instance, oceans





Oleander campaign



Zonal (solid) and meridional (dashed–dotted) velocity spectra from the Oleander observations. Dashed lines indicate a 23 slope. The 95% confidence interval is marked (from Wang et al., JPO, 2010)

Structure functions

- Locally homogeneous and isotropic turbulence can be analyzed using structure functions
- Velocity increments are computed along a vector **L** joining two points separated by a distance r
- Parallel and orthogonal projections of the velocity increments upon **L** are longitudinal (L) and transverse (T) - $\delta u_L(r)$ and $\delta u_T(r)$
- Statistical moments of the velocity increments are structure functions:

$$D_{LL}(r) = <\delta u_L \delta u_L>, \qquad D_{TT}(r) = <\delta u_T \delta u_T>$$

- In isotropic and homogeneous turbulence, d – dimension of space,

$$D_{TT}(r) = D_{LL}(r) + \frac{r}{d-1}\frac{dD_{LL}(r)}{dr}$$

- On large scales, $D_{TT}(r)/D_{LL}(r)$ = 3 in 2D and 2 in 3D flows
- QNSE gives 2.59 and so 2 < d=2.26 < 3!

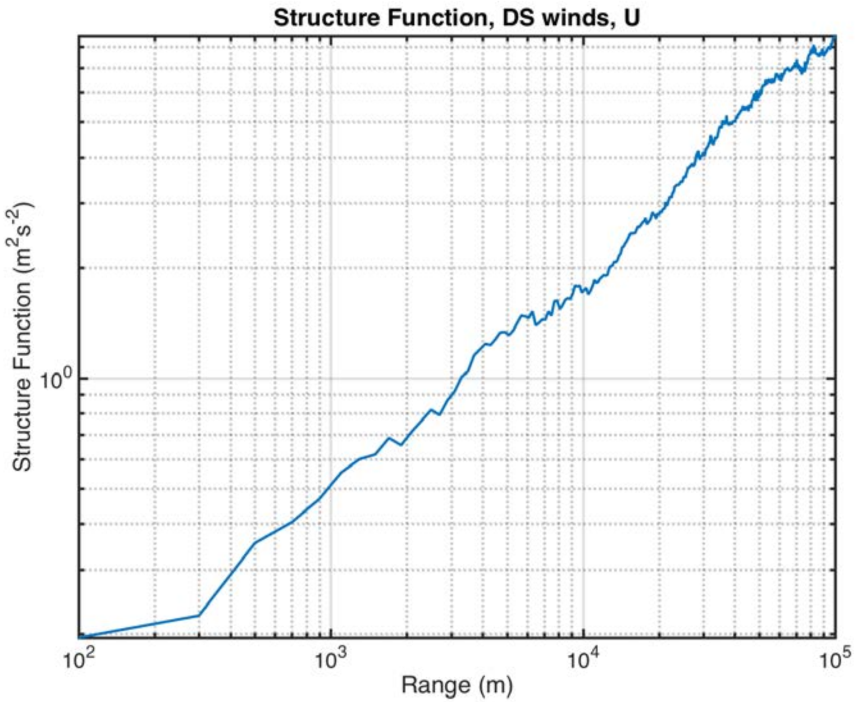
- Wiener-Khinchin relations:

$$D_{LL}(r) = 2\int_0^\infty (1-\cos kr)E_L(k)dk,$$

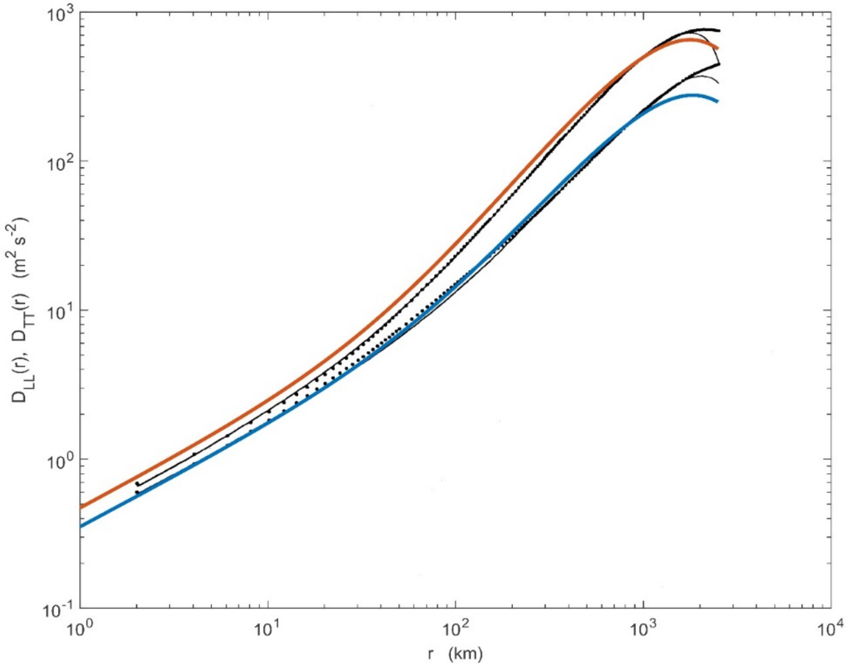
$$D_{TT}(r) = 2\int_0^\infty (1-\cos kr)E_T(k)dk.$$

- 1D spectra are known from QNSE ➔ compute structure functions and compare with data

Rodriguez & Wineteer (2018)



MOSAIC data (Lindborg, 1999)



Conclusions

- Turbulence and Kolmogorov cascades are ubiquitous
- Typical for planetary systems are anisotropic turbulence and dispersive waves, universal features
- QNSE theory yields analytical expressions for universal spectra
- Rotation suppresses turbulence degrees of freedom, renders turbulence dimensionality >2 but <3
- Hints to fractal dimension and large-scale intermittency ➔ storm events; further theoretical derivations are necessary
- Scatterometry-established structure functions combined with QNSE results can be used to establish ε in the atmosphere and oceans simultaneously – significant improvement in the performance of circulation models can be expected
- Quantitative framework for scale-dependent eddy viscosities, hyperviscosities, diffusivities, etc.
- ***Major conclusions:***
- ***Future belongs to satellite methods – extract spectral characteristics from physical space data***
- ***Need a good theory to understand and quantify the physics***
- ***Need to replace concepts of geostrophic turbulence and quasi-two-dimensionalization by suppression of turbulence degrees of freedom by rotation***