How QNSE theory and scatterometry can improve various data products

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Turbulence is all around us

• Campaigns on dispersion in the atmosphere and ocean reveal Richardson diffusion regime – strong evidence of Kolmogorov turbulence
• May feature either direct or inverse cascade
• If we know the rate of the cascade, $\varepsilon$, we can compute eddy viscosity and eddy diffusivity (mixing coefficients) even in anisotropic turbulence
• Estimating mixing coefficients is one of the most difficult problems for NWP, modeling of geophysical and planetary circulations
• Mixing coefficients should recognize the interaction between turbulence and waves
• Three main types of waves: internal (N), inertial (f), Rossby ($\beta$)
• What are controlling scales that involve turbulence ($\varepsilon$) and waves?

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Controlling (crossover) scales

• Ozmidov $L_\Omega = (\varepsilon/N^3)^{1/2}$
• Woods $L_\Omega = (\varepsilon/f^3)^{1/2}$
• Transitional $L_\beta = (\varepsilon/\beta^3)^{1/5}$
• Advantageous over $|\zeta|/f$ which does not delineate length scales
• On smaller scales, turbulence prevails → Kolmogorov spectra
• On larger scales, wave-related parameters dominate, spectra become universal
• Can be used as performance outliers
Universal vertical spectrum of the horizontal velocity

\[ E_1(k_3) = 0.626 \varepsilon^{2/3} k_3^{-5/3} + 0.214 N^2 k_3^{-3} \]

\[ = 0.626 \varepsilon^{2/3} k_3^{-5/3} \left[ 1 + 0.34(k_3/k_0)^{-4/3} \right] \]

Gregg, Winkel, Sanford, JPO, 1993

Smith, Fritts, Van Zandt, JAS, 1987

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Universal horizontal spectrum of the horizontal velocity

- Upper troposphere and lower stratosphere – Global Atmospheric Sampling Program (GASP; Nastrom & Gage, 1985), Measurement of Ozone by Airbus In-Service aircraft (MOSAIC; Marenco, 1998, Lindborg, 1999)
- Nastrom & Gage spectra, or canonical spectra
  - On synoptic scales (2-3 x 10^3 to 500km), the slope is $k_h^{-3}$, $k_h$ – the horizontal wavenumber
  - On mesoscales (500 to 10 km), spectrum transitions to the Kolmogorov $k_h^{-5/3}$ slope
  - Spectra are remarkably universal throughout the troposphere and stratosphere, the seasons, but are dependent on latitude
  - Spectral amplitude decreases towards the equator

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Zonal and residual spectra on Jupiter from Cassini data

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The physics of the Nastrom & Gage spectrum

• Still no consensus

• Most common – $k_h^{-3}$ branch is due to direct enstrophy cascade, following Kraichnan’s theory of 2D turbulence and Charney’s (1971) scaling

• But – O’Gorman & Schneider (2007) showed that enstrophy cascade is unnecessary

• Cho & Lindborg (2001) – there is no explanation of the $k_h^{-3}$ range other than Charney’s two-dimensionalization

• Lovejoy (2009) – $k_h^{-3}$ range is not present; is due to analysis errors

• Dynamics of $k_h^{-5/3}$ range is even less clear

• Was considered to be due to inverse cascade – but Lindborg (1999) used structure functions to demonstrate direct cascade

• Lindborg suggested stratified turbulence – but Skamarock et al. (2014) showed it unsupported

• Cho & Lindborg (2001) – the Coriolis force may be crucial but nearly impossible to account for via the energy equation

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With no clear understanding of the physics, the N&G spectrum has been used as an outlier

- The dynamics has implications for forecast error growth, spinup time scales, filtering and subfilterscale physics (e.g., Skamarock 2004; Shutts 2005)
- Atmospheric models’ ability to reproduce N&G spectrum serves as validation of the correctness of model’s formulation, implementation, and configuration
- KE spectrum provides a measure of model’s effective resolution and filter effects
- Lovejoy et al. (2009) brought up the importance of turbulence and its anisotropization in interpretation of the N&G spectra
- Suggested that in anisotropic turbulence, structures progressively flatten out with increasing scale and may obey a power law that obliterates the need in invoking the 3D-2D transition
- ‘the entire mainstream view of the atmosphere has fundamentally been coloured by the assumption of isotropic turbulence’

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Yano (2010):

“…My naive feeling is that an elaborated use of a renormalization group (RNG) theory might potentially lead to a necessary theoretical breakthrough, but I should not be too speculative.”

He was not too speculative…

RNG-based Quasi-Normal Scale Elimination (QNSE) theory indeed offers a breakthrough

- Yields analytical expression for the N&G spectrum and explains its major features, both qualitatively and quantitatively
- Considers 3D fluid occupying infinite domain; dynamics is represented by full Navier-Stokes and continuity equations in a coordinate frame rotating with the angular velocity $\Omega$
- QNSE is an algorithm of successive coarsening of the flow domain by cyclically eliminating small shells of wave number modes and computing compensating corrections to the viscosity
- No quasi-geostrophy is implied
Mathematical formulation

\[
\frac{\partial v_\alpha}{\partial t} + (\mathbf{v} \cdot \nabla) v_\alpha + (\mathbf{f} \times \mathbf{v})_\alpha = v_0 \nabla^2 v_\alpha - \frac{\partial P}{\partial x_\alpha} + \xi_0^0
\]

\[
\frac{\partial v_\alpha}{\partial x_\alpha} = 0
\]

Space-time Fourier transform (d=3 is the dimension of space):

\[
v_\alpha (\mathbf{x}, t) = \frac{1}{(2\pi)^{d+1}} \int_{k \leq k_d} \int d\omega v_\alpha (\omega, \mathbf{k}) \exp[i(\mathbf{kx} - \omega t)]
\]
The crossover between turbulence and inertial waves is on scales $O(L_\Omega)$

$L_\Omega = (\varepsilon/f^3)^{1/2}$ is the Woods scale, $f = 2\Omega$ is the Coriolis parameter

Rotation leads to the development of the inverse cascade on scales $> L_\Omega$

The procedure of the coarse-graining:

- Introduce the dynamic dissipation cutoff wavenumber, $\Lambda$, a small shell $\Delta \Lambda$, $\Delta \Lambda / \Lambda << 1$, `slow' and `fast' modes
- Compute $O(\Delta \Lambda)$ correction to the inverse Green function by ensemble averaging of the fast modes over $\Delta \Lambda$. This correction generates $O(\Delta \Lambda)$ accruals to all renormalized viscosities while preserving the analytical form of the governing equations
- All viscosities are updated and the process moves forward towards elimination of the next shell $\Delta \Lambda$
The analytical solution

\[ \frac{\nu_h}{\nu_n} = 1 - \frac{41}{252} Ro(k)^{-2} = 1 - 0.77 (k / k_\Omega)^{-4/3} \]

\[ \frac{\nu_z}{\nu_n} = 1 - \frac{73}{1260} Ro(k)^{-2} = 1 - 0.27 (k / k_\Omega)^{-4/3} \]

\[ \frac{\nu_3}{\nu_n} = 1 - \frac{37}{1260} Ro(k)^{-2} = 1 - 0.14 (k / k_\Omega)^{-4/3} \]

\[ \frac{\nu_{3z}}{\nu_n} = 1 + \frac{19}{252} Ro(k)^{-2} = 1 + 0.36 (k / k_\Omega)^{-4/3} \]

\[ \frac{\kappa_h}{\nu_n} = \alpha + 0.049 Ro(k)^{-2} = \alpha + 0.23 (k / k_\Omega)^{-4/3} \]

\[ \frac{\kappa_z}{\nu_n} = \alpha - 0.001 Ro(k)^{-2} = \alpha - 0.0049 (k / k_\Omega)^{-4/3} \]

\[ \alpha = \kappa_n / \nu_n = Pr_t^{-1} \approx 1.39 \quad \text{the inverse turbulent Prandtl number in non-rotating flows} \]
Viscosity and diffusivity renormalization

• In turbulence on f-plane introduce the Woods scale
  \[ L_\Omega = \left( \frac{\varepsilon}{f^3} \right)^{1/2} \]

• Analogous to Ozmidov scale

• Weak rotation, \( k/k_\Omega = O(1) \)
• On small scales, turbulence is isotropic and Kolmogorov-like
• Viscosity undergoes anisotropization and componentality
• 4 renormalized viscosities that act in different directions and on different velocity components
• Horizontal viscosity \( \rightarrow 0 \Rightarrow \) indication of the inverse cascade
Nastrom & Gage Spectra

• Zonal spectrum of zonal velocity (longitudinal):

\[ E_1(k) = 0.47 \epsilon^{2/3} k_1^{-5/3} + 0.0926 f^2 k_1^{-3} \]

• The dissipation rate \( \epsilon = 5 \times 10^{-5} \text{ m}^2\text{s}^{-3} \)
  (after Frehlich & Sharman, 2010)

• Latitude 30°N

• Transverse spectrum:

\[ E_2(k) = 0.626 \epsilon^{2/3} k_1^{-5/3} + 0.240 f^2 k_1^{-3} \]

• The flow is forced on large scales and features direct cascade throughout to small scales

• The physics: Anisotropic turbulence with dispersive (inertial) waves

• Spectra are universal as they depends on \( f \) only
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Dependence of the spectrum on latitude

- Has not been explained
- QNSE provides quantitative explanation
- In good agreement with data
- Suggests that the $k_h^{-3}$ branch may completely disappear near the equator
- Simulations with $f=0$ (e.g., Durran et al., MWR, 2017; Weyn & Durran, JAS, 2017; Sun et al., JAS, 2017) yield $k_h^{-5/3}$ spectra only
- Directly observed in the data for the ocean near-surface winds (Xu et al., 2011)
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Ocean campaigns

• If the large-scale spectrum depends on $f$ only, one expects to find it in other environments, for instance, oceans.
Oleander campaign

Zonal (solid) and meridional (dashed–dotted) velocity spectra from the Oleander observations. Dashed lines indicate a 23 slope. The 95% confidence interval is marked (from Wang et al., JPO, 2010)
Structure functions

• Locally homogeneous and isotropic turbulence can be analyzed using structure functions

• Velocity increments are computed along a vector \( \mathbf{L} \) joining two points separated by a distance \( r \)

• Parallel and orthogonal projections of the velocity increments upon \( \mathbf{L} \) are longitudinal (L) and transverse (T) - \( \delta u_L(r) \) and \( \delta u_T(r) \)

• Statistical moments of the velocity increments are structure functions:

\[
D_{LL}(r) = \langle \delta u_L \delta u_L \rangle, \quad D_{TT}(r) = \langle \delta u_T \delta u_T \rangle
\]

• In isotropic and homogeneous turbulence, \( d \) – dimension of space,

\[
D_{TT}(r) = D_{LL}(r) + \frac{r}{d-1} \frac{dD_{LL}(r)}{dr}
\]

• On large scales, \( D_{TT}(r)/D_{LL}(r) = 3 \) in 2D and 2 in 3D flows

• QNSE gives 2.59 and so \( 2 < d < 2.26 < 3 \! \)!
• Wiener-Khinchin relations: 
  \[ D_{LL}(r) = 2\int_0^{\infty} (1 - \cos kr)E_L(k)dk, \]
  \[ D_{TT}(r) = 2\int_0^{\infty} (1 - \cos kr)E_T(k)dk. \]

• 1D spectra are known from QNSE \(\rightarrow\) compute structure functions and compare with data

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Rodriguez & Wineteer (2018)  
MOSAIC data (Lindborg, 1999)

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Conclusions

• Turbulence and Kolmogorov cascades are ubiquitous
• Typical for planetary systems are anisotropic turbulence and dispersive waves, universal features
• QNSE theory yields analytical expressions for universal spectra
• Rotation suppresses turbulence degrees of freedom, renders turbulence dimensionality >2 but <3
• Hints to fractal dimension and large-scale intermittency → storm events; further theoretical derivations are necessary
• Scatterometry-established structure functions combined with QNSE results can be used to establish $\varepsilon$ in the atmosphere and oceans simultaneously – significant improvement in the performance of circulation models can be expected
• Quantitative framework for scale-dependent eddy viscosities, hyperviscosities, diffusivities, etc.

• **Major conclusions:**
• *Future belongs to satellite methods – extract spectral characteristics from physical space data*
• *Need a good theory to understand and quantify the physics*
• *Need to replace concepts of geostrophic turbulence and quasi-two-dimensionalization by suppression of turbulence degrees of freedom by rotation*