Some thoughts on the resolution of satellite sampling patterns, with application to QuikSCAT

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An estimate of the zonal-wavenumber/ frequency spectrum of meridional wind stress in the equatorial Pacific:

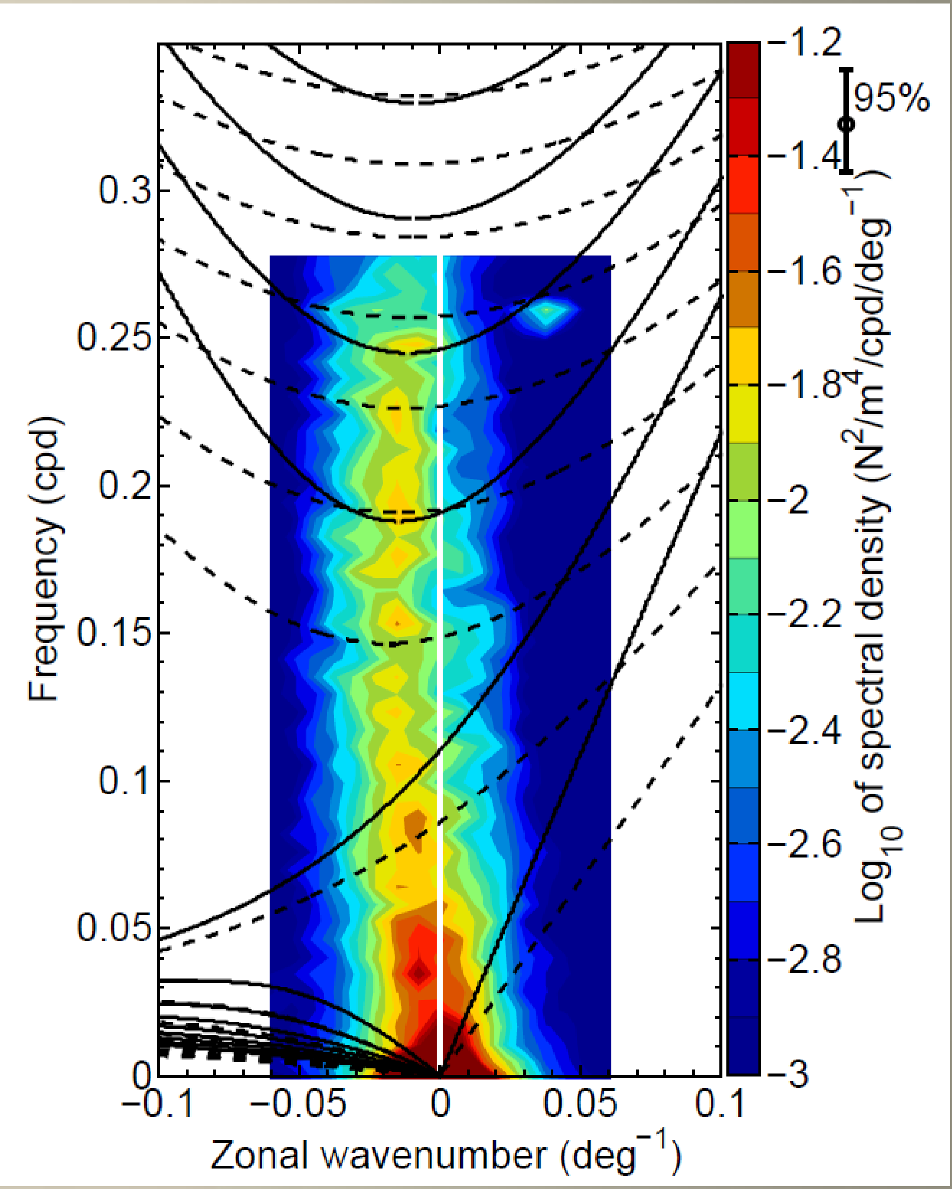
Approach:

→ Grid the data in time-longitude using loess smoother; then estimate Fourier transform by FFT using 6 years of data (2000-2005).

→ Form spectra and average over latitude (8°S-8°N).

Data: Ascending/descending pass QuikSCAT winds from Remote Sensing Systems (Ku-2011 GMF)

Meridional wind stress from QuikSCAT (2000-2005)



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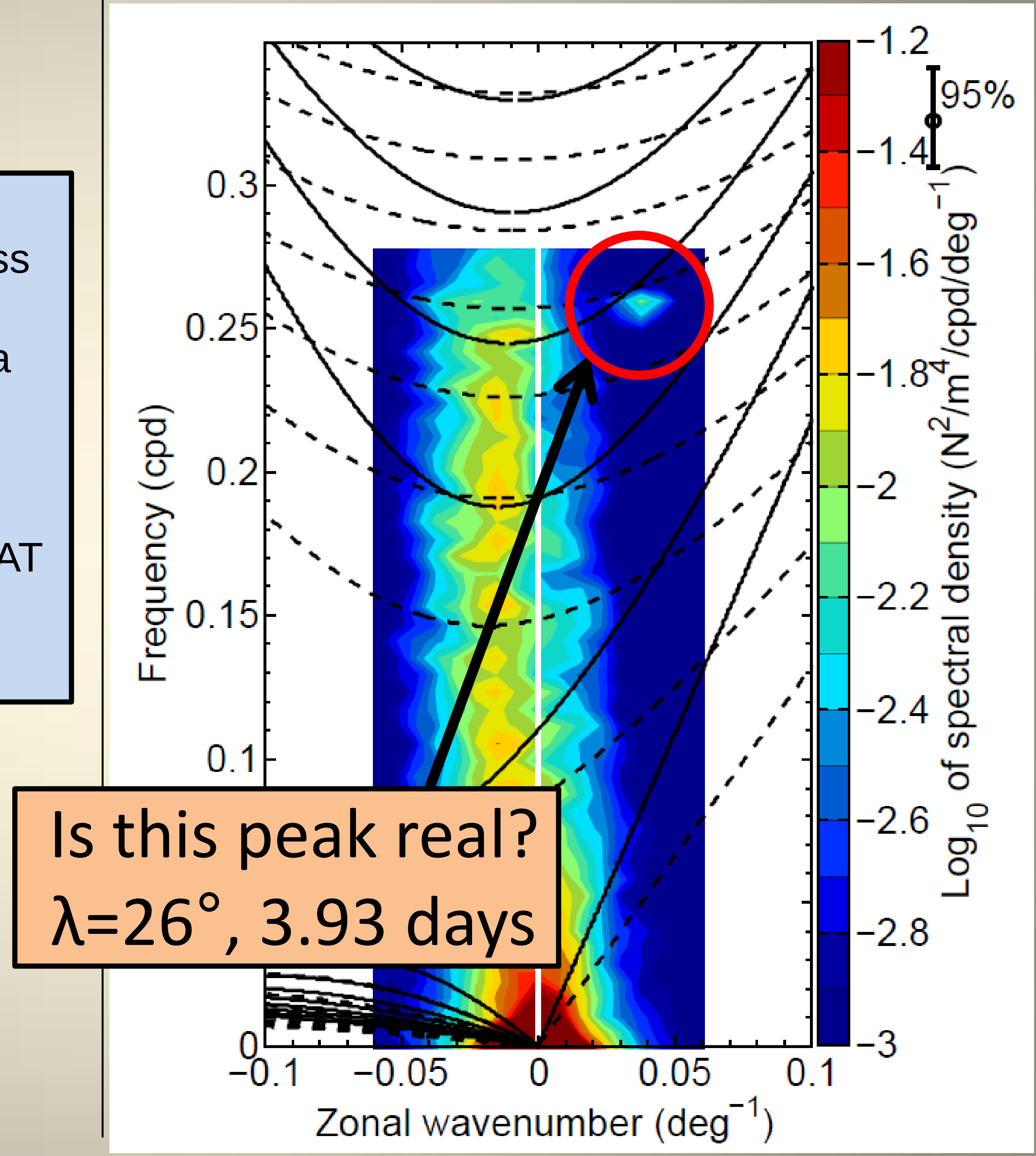
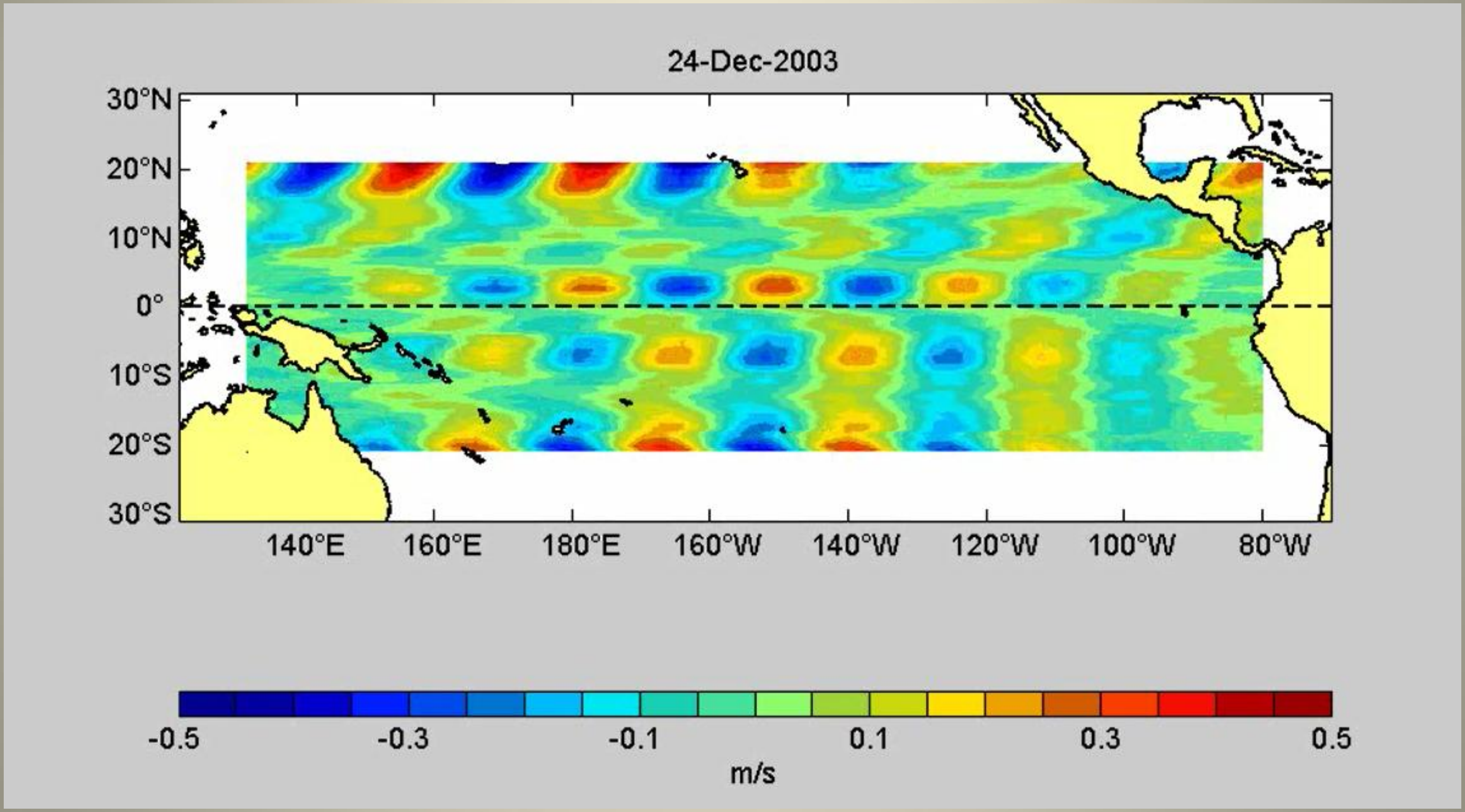
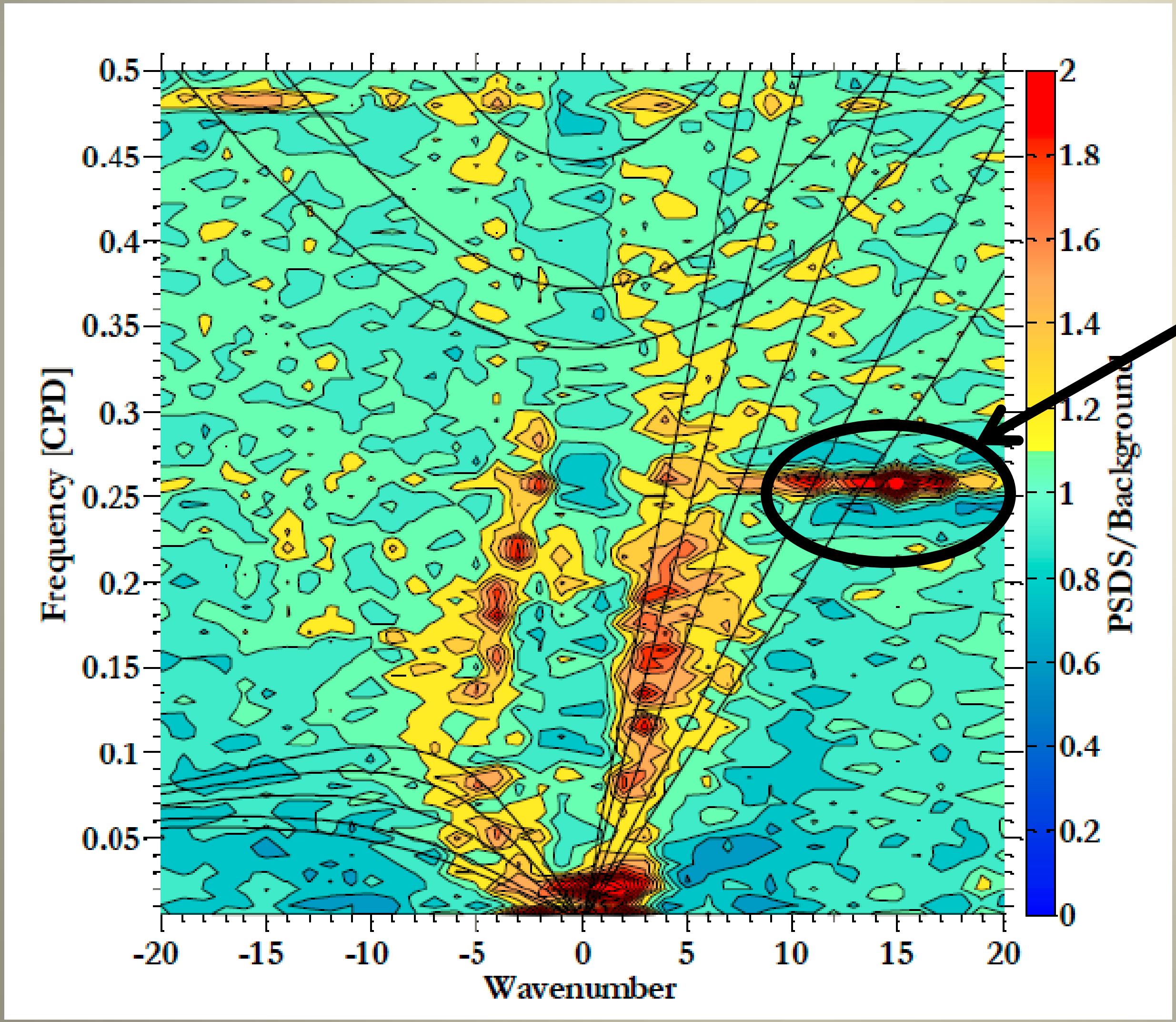


Meridional wind bandpass filtered around the eastward-propagating 4-day, 26°-wavelength band



Zonal-wavenumber/ frequency spectrum of IFREMER 'daily' QuikSCAT product (Bentamy and Croizé-Fillon, 2012) (Figure from Jerome Patoux's 2012 OVWST talk)



Spectrum is normalized by a background spectrum, but *the same peak is present in the IFREMER product*

How can we decide whether a spectral band is *resolved*?

How can we assess which wavenumbers and frequencies of the true variability might contaminate a particular band of our estimate?

→ Taken as a whole, the literature on satellite resolution is confusing.
- → It is not obvious how to use any of the existing approaches to answer the questions posed above.
- → It is not obvious how the various approaches are related to one another.

→ Although some of you may already know why this peak is there, ***every*** band of these spectral estimates contains alias contributions. We'll use this particular alias as an example.

→ When we talk about resolution, we generally are asking how well we can determine the Fourier coefficients from the data

$$\boxed{\hat{\boldsymbol{\alpha}} = \boldsymbol{\Gamma}\mathbf{y}}$$

Vector of estimated Fourier coefficients (Mx1)

Whatever linear operations we perform to get from raw data to an estimate of the Fourier coefficients

Vector of sampled data (Nx1)

→ So, we are making some particular estimate, $\hat{\boldsymbol{\alpha}}$, of the Fourier coefficients using some particular set of linear operations, Γ,on the measured data, $\mathbf{y}$:

$$\boxed{\hat{\boldsymbol{\alpha}} = \boldsymbol{\Gamma}\mathbf{y}}$$

→ The (error-free) samples are the inverse Fourier transform of the true Fourier transform:

$$y_n = \int_{-\infty}^{\infty} \alpha(f) e^{i2\pi f t_n} \mathrm{d}f \xrightarrow[K\to\infty,\, \Delta f\to 0]{\text{Discretize}} y_n = \sum_{k=1}^{K} \alpha_k e^{i2\pi f_k t_n}$$

That is, $\boxed{\mathbf{y} = \mathbf{E}\boldsymbol{\alpha}}$ $\mathbf{E}$ is an NxK inverse Fourier transform matrix (#samples by #elements in true spectrum)

→ Now we have our answer, relating each element of our estimated spectrum to each element of the true spectrum:

$$\boxed{\hat{\boldsymbol{\alpha}} = \boldsymbol{\Gamma}\mathbf{E}\boldsymbol{\alpha}}$$

→ The matrix $\mathbf{\Gamma E}$ tells us how our estimate is a specific linear combination of the true Fourier coefficients:

$$\boxed{\hat{\boldsymbol{\alpha}} = \mathbf{\Gamma E}\boldsymbol{\alpha}}$$

→ In the context of general linear estimates, $\mathbf{\Gamma E}$ is known as the "hat" matrix, and it is very closely related to the "alias" or "bias" matrix (e.g., Draper and Smith's textbook, *Applied Regression Analysis*)

→ Assuming the relative phases of the true Fourier coefficients are random, we can express this in terms of the estimated and true spectra (not Fourier coefficients):

$$\boxed{\left\langle \left|\hat{\boldsymbol{\alpha}}\right|^2 \right\rangle = \mathbf{R}\left|\boldsymbol{\alpha}\right|^{\mathbf{2}}}$$ where $\mathbf{R} = \left|\mathbf{\Gamma E}\right|^2$ and $\left|\ \right|^2$ is element-wise

This "resolution matrix" **R** is the basic result

$$\boxed{\left\langle \left|\hat{\boldsymbol{\alpha}}\right|^2 \right\rangle = \mathbf{R}\left|\boldsymbol{\alpha}\right|^2}$$

Vector of estimated spectral values

Vector of true spectral values

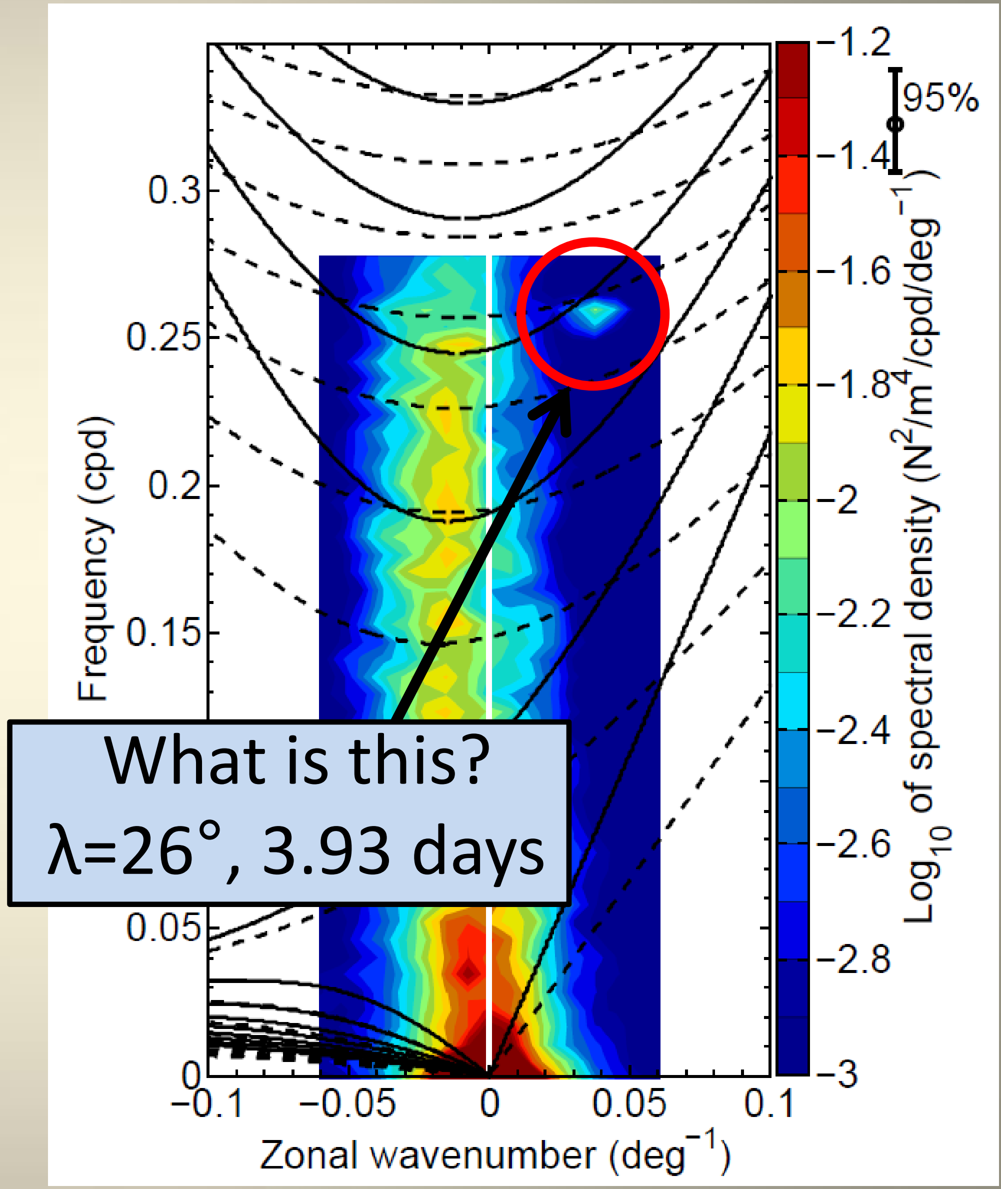
<u>**R** gives us two powerful tools:</u>

→ A row of **R** tells us how each frequency of the true spectrum influences our estimate at a particular frequency (a "transfer function")

→ A column of **R** tells us the spectrum we would estimate if the true spectrum contained only a single frequency (a "response function")

To do the resolution analysis, we just need to write down the linear operations we performed to estimate the Fourier coefficients from the swath data:

Meridional wind stress from QuikSCAT (2000-2005)



To do the resolution analysis, we just need to write down the linear operations we performed to estimate the Fourier coefficients from the swath data:

(1) We gridded the data (in lon-time).
(2) We applied a taper window to the gridded data.
(3) We computed the discrete Fourier transform.

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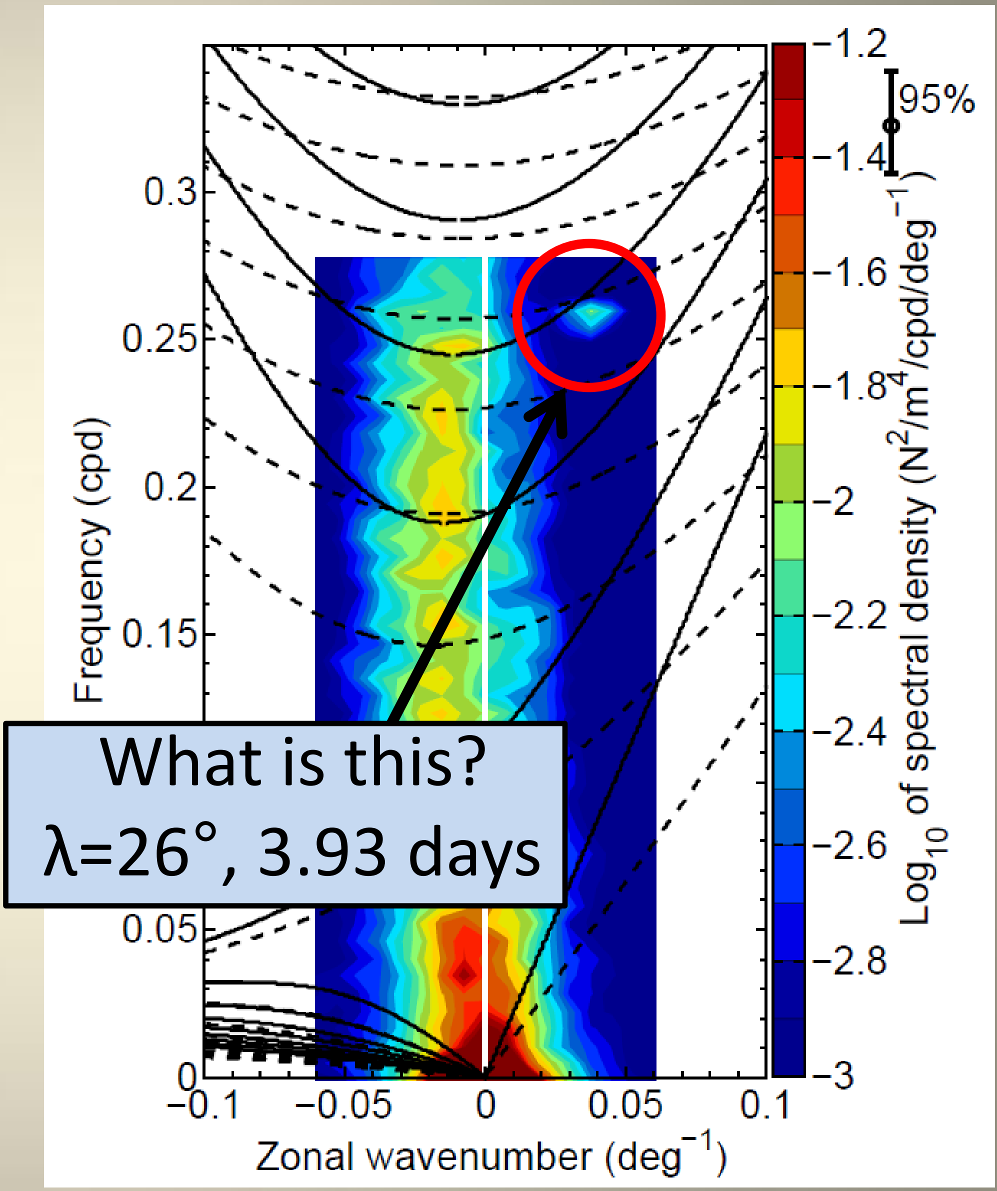

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$$\hat{\boldsymbol{\alpha}} = \mathbf{F}\mathbf{W}_2\mathbf{W}_1\mathbf{y} = \mathbf{F}\mathbf{W}_2\mathbf{W}_1\mathbf{E}\boldsymbol{\alpha}$$

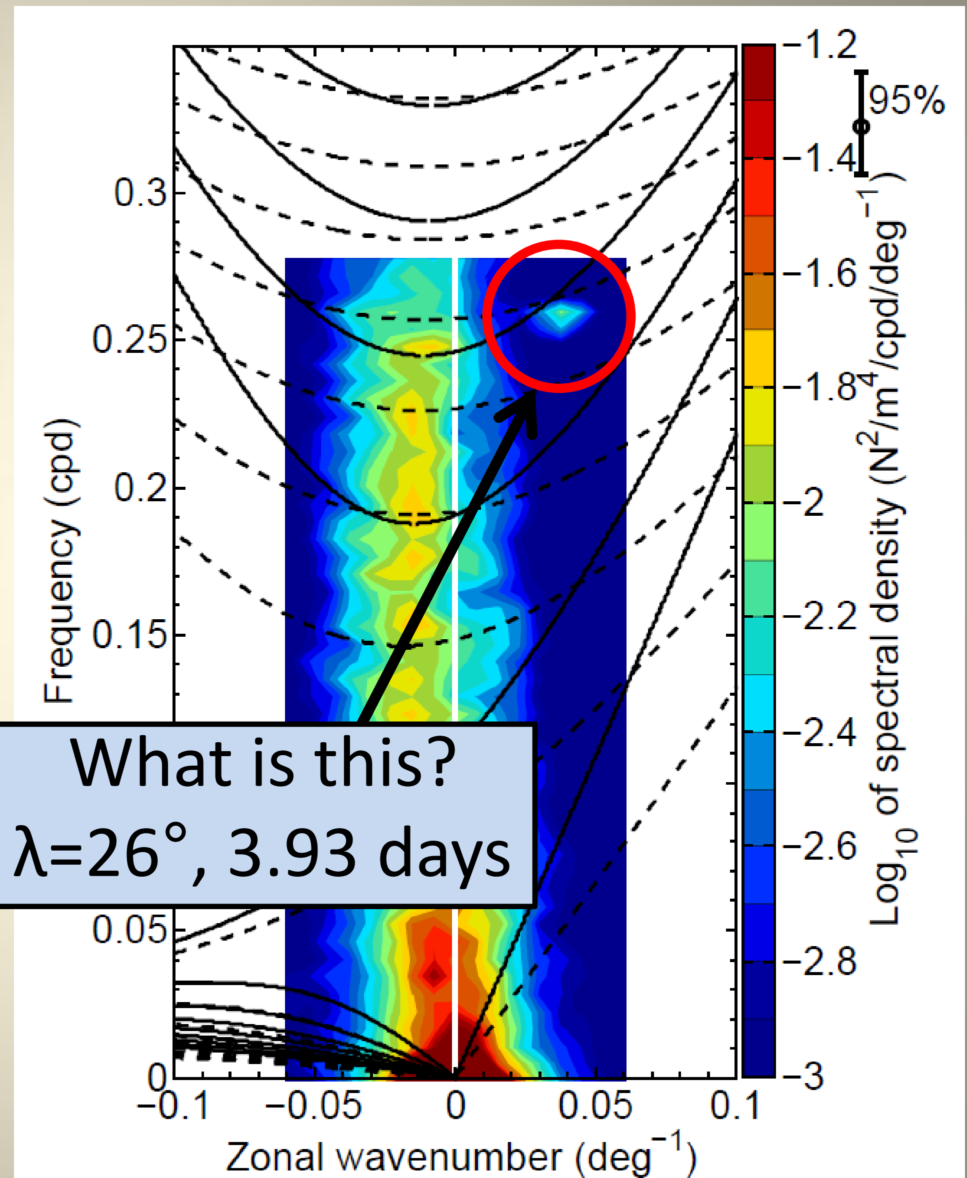
Fourier transform

taper window

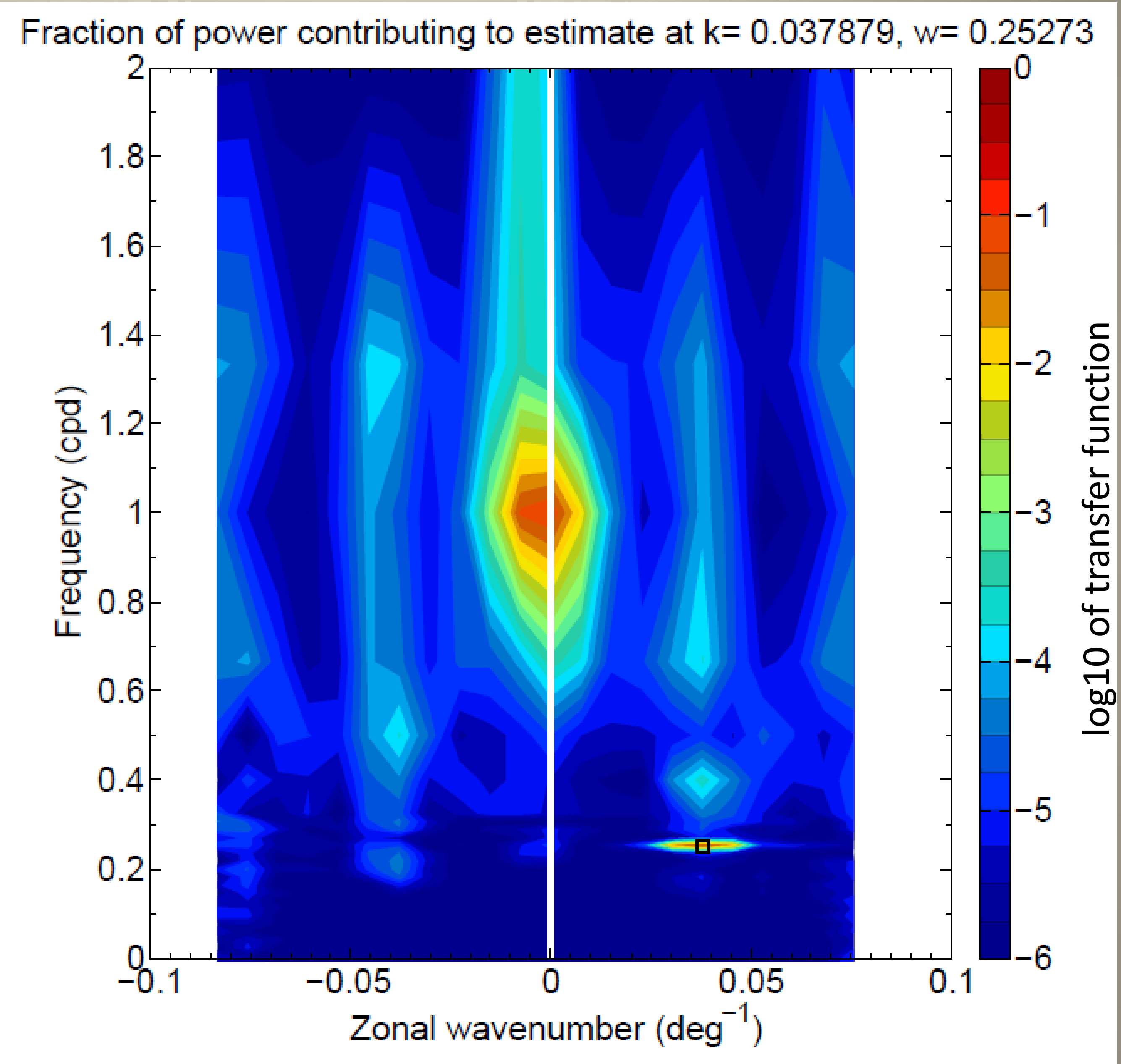
gridding

$$\boxed{\mathbf{R} = \left|\mathbf{F}\mathbf{W}_2\mathbf{W}_1\mathbf{E}\right|^2}$$

Meridional wind stress from QuikSCAT (2000-2005)



Row of resolution matrix: tells us the parts of the spectrum contributing to the “alias” peak in our estimate



Our estimate responds strongly to diurnal signals of global scale that propagate westward, passing ~10% of the variance (i.e., ~30% of the amplitude)

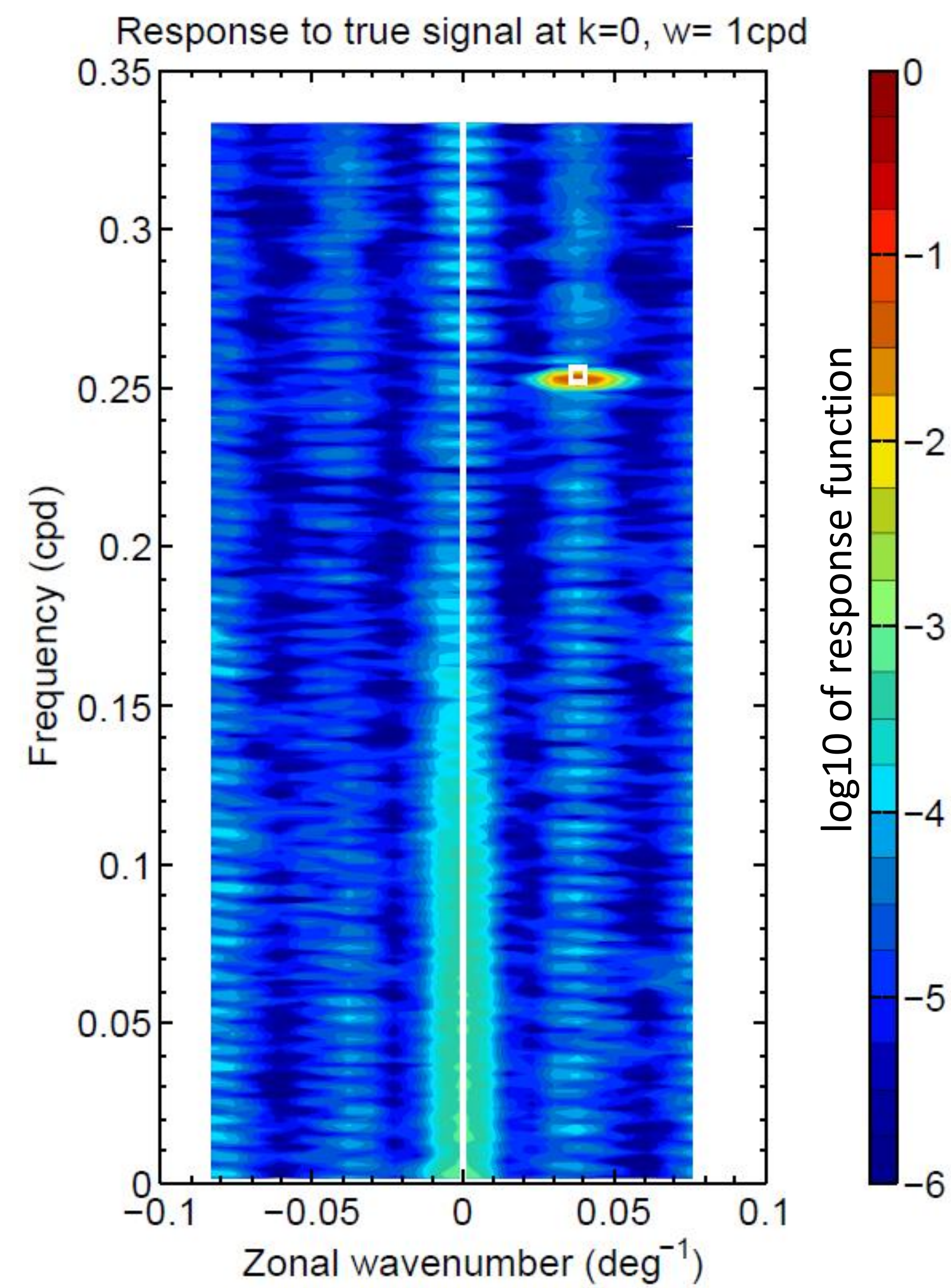
Column of resolution matrix: tells us how each band in our estimate responds to a global-scale diurnal signal

What other bands are contaminated by the diurnal cycle?

Besides the major alias peak we could see in the spectrum, the worst contamination is at basin scales and periods exceeding 7 days, but the alias is ~100 times weaker.

The total variance of the aliasing to low frequencies is comparable, though, because there are lots affected bands (~110 bands).

About 50% of the variance of the global-scale diurnal cycle appears spuriously in my gridded field!



Conclusions:

(1) There is a simple framework we can use to analyze the resolution of a spectrum or gridded field made from satellite data.

(2) Aliasing of the global-scale diurnal cycle by the QuikSCAT sampling is an appreciable source of error in two gridded QuikSCAT products, and probably in others, too.

(3) Not shown: other existing approaches to analyzing resolution can be easily derived in this framework