Influence of Group Velocity on the Relative Distributions of Equatorial Wind Forcing and Oceanic Response

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Zonal-wavenumber/ frequency spectrum of SSH from fit to along-track altimeter data in the equatorial Pacific Ocean (averaged 6°S-6°N, 2000-2005)

Solid dispersion curves: baroclinic mode 1

Dashed dispersion curves: baroclinic mode 2

(Comparable spectra of TAO-TRITON dynamic height in Farrar and Durland, JPO, Nov, 2012)
4 day period

10 day period
Preliminary comparisons of wind stress and SSH spectra, from QuikSCAT and altimetry

Narrow banded, low-wavenumber nature of atmospheric forcing limits oceanic resonances to low wavenumbers.

Deduced by Wunsch and Gill, 1976, from very little data.

We finally have sufficient observations to verify their conjecture.
Is the energy in these peaks reduced by the ability to leave a forcing region when the group velocity does not vanish?
Forced and damped non-dimensional equations

\[-i\sigma u - yv + ikp = 0\]
\[-i\sigma v + yu + p_y = Y_s\]
\[-i\sigma p + iku + v_y = 0\]

\(Y_s \propto\) meridional wind stress that is symmetric about equator
\(\sigma \equiv \omega + i\epsilon: \) complex frequency
\(\epsilon: \) inverse dissipation time scale

Symmetric meridional wind stress can only force even numbered meridional modes with \(p\) (SSH) that is anti-symmetric about the equator.
Mode 2 dispersion curve (black)

Normalized response to $X = 0$, $Y = \psi_2(y)$, broadband in $k$, $\varepsilon = 0.05$
Mode 2 dispersion curve

Normalized response to $X = 0$, $Y = \psi_2(y)$, broadband in $k$, $\varepsilon = 0.05$

Wavenumber spectrum of narrow-banded forcing that is isolated in space
Approximations for very narrow band forcing

All forcing components in phase

\[ (v_m, u_m, p_m) \approx v_m (V_m, U_m, P_m) \] (Known meridional structures of free-wave velocity and pressure)

\[ \langle u_m p_m \rangle \approx E_m c_{gx} \] (Energy density times zonal group velocity of free wave)

\[ E_m \equiv \left| v_m \right|^2 \left\langle \left| V_m \right|^2 + \left| U_m \right|^2 + \left| P_m \right|^2 \right\rangle \propto \left\langle \left| P_m \right|^2 \right\rangle \] (Total wave energy density)

\[ \langle \rangle \quad \text{: Period average} \]

\[ \langle \rangle \quad \text{: Meridional integral} \]
Simplified energy equation for meridional mode m

\[ L_\varepsilon \frac{d}{dx} E_m + E_m = \gamma \left\langle V_{(m)}(y) \, Y_s(x, y) \right\rangle \]

where \( L_\varepsilon = c_{gx} / 2\varepsilon \) is the energy dissipation length scale.

Solution:

\[ E_m(x) = L_\varepsilon^{-1} \int_{0}^{\infty} \exp(-\xi / L_\varepsilon) \left\langle V_{(m)}(y) \, Y_s(x - \xi, y) \right\rangle d\xi \]
Isolated forcing at frequency where oceanic group velocity = 0.3 (nondim)

\[ \varepsilon = 0.10 \quad L_\varepsilon = 1.5 \]

\[ \varepsilon = 0.05 \quad L_\varepsilon = 3 \]

thin: Fourier transform of response to broadband forcing
thick: Fourier transform of narrow-band isolated forcing

thick black: Energy solution using Fourier transforms
dashed red: 1st-order model solution
dashed black: Energy solution w/ no group velocity
Isolated forcing at frequency where oceanic group velocity = 0.3 (nondim)

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boundary
bc mode 1, merid mode 2

4–4.1 day band,
\[ c_{gx} \approx 0.15 \text{ deg / day} \]
(20 cm/s)

bc mode 1, merid mode 0

9.3–9.7 day band,
\[ c_{gx} \approx 1.0 \text{ deg / day} \]
(130 cm/s)

6 year record: 10 frequency bands in each of the above bands
log_{10} antisymmetric SSH spectral density for 9.3 − 9.7 days

Squared coherence vs 3N 160W for 9.3 − 9.7 days

log_{10}, square of coherence gain fit to $P(x,y)$ for bc1m0
log\(_{10}\) antisymmetric SSH spectral density for 9.3 – 9.7 days

log\(_{10}\), square of coherence gain fit to \(P(x,y)\) for bc1m0

Integrated antisymmetric SSH variance. Black: total, Red: coherent bc1m0 (69%)
\( \log_{10} \) symmetric \( \tau^y \) spectral density for 9.3 – 9.7 days

Symmetric \( \tau^y \) coherence gain versus SSH at 3 N, 160 W

Normalized \( V^{(0)} \tau^y \) coherent with SSH at 3 N, 160 W
bc1m0, 9.5 days, $c_g = 1.0$ deg/day (1.3 m/s)

normalized $V_{(0)}^{\tau^y}$ coherent with SSH at 3 N, 160 W

coherence gain$^2$ SSH, fit to $P^2(x,y)$

$\langle V_{(0)}^{\tau^y} \rangle$, $\langle \text{Gain}^2_{\text{SSH}} \rangle$
bc1m0, 9.5 days, $c_g = 1.0$ deg/day (1.3 m/s)

normalized $V_{(0)}^\tau$ coherent with SSH at 3 N, 160 W

coherence gain$^2$ SSH, fit to $P^2(x,y)$

$\langle V_{(0)}^\tau \rangle$, $\langle \text{Gain}^2_{\text{SSH}} \rangle$, conv[$(e^{-x/L_\varepsilon}/L_\varepsilon)$, $\langle V^\tau \rangle(x)$]
bc1m2, 4.0 days, $c_g = 0.2$ deg/day (0.2 m/s)

normalized $V_{(2)}^\tau$ coherent with SSH at 2 N, 140 W

coherence gain $^2$ SSH, fit to $P^2(x,y)$

$\langle V_{(2)}^\tau \rangle$, $\langle \text{Gain}^2_{\text{SSH}} \rangle$, $\text{conv}[e^{-x/L_\varepsilon} / L_\varepsilon$, $\langle V^\tau \rangle(x)]$

$L_\varepsilon = 4$ deg
$T = 24$ days
Tentative Conclusions

1) Group velocity displaces oceanic response “downwind” of forcing by dissipation length scale
2) If displaced response remains within basin, energy not lost from mode
3) In the 4-10 day peaks we see, it doesn’t look like much energy is lost because of group velocity
4) Dissipation time scale can be estimated from forcing-response displacement