Influence of Group Velocity on the Relative Distributions of Equatorial Wind Forcing and Oceanic Response

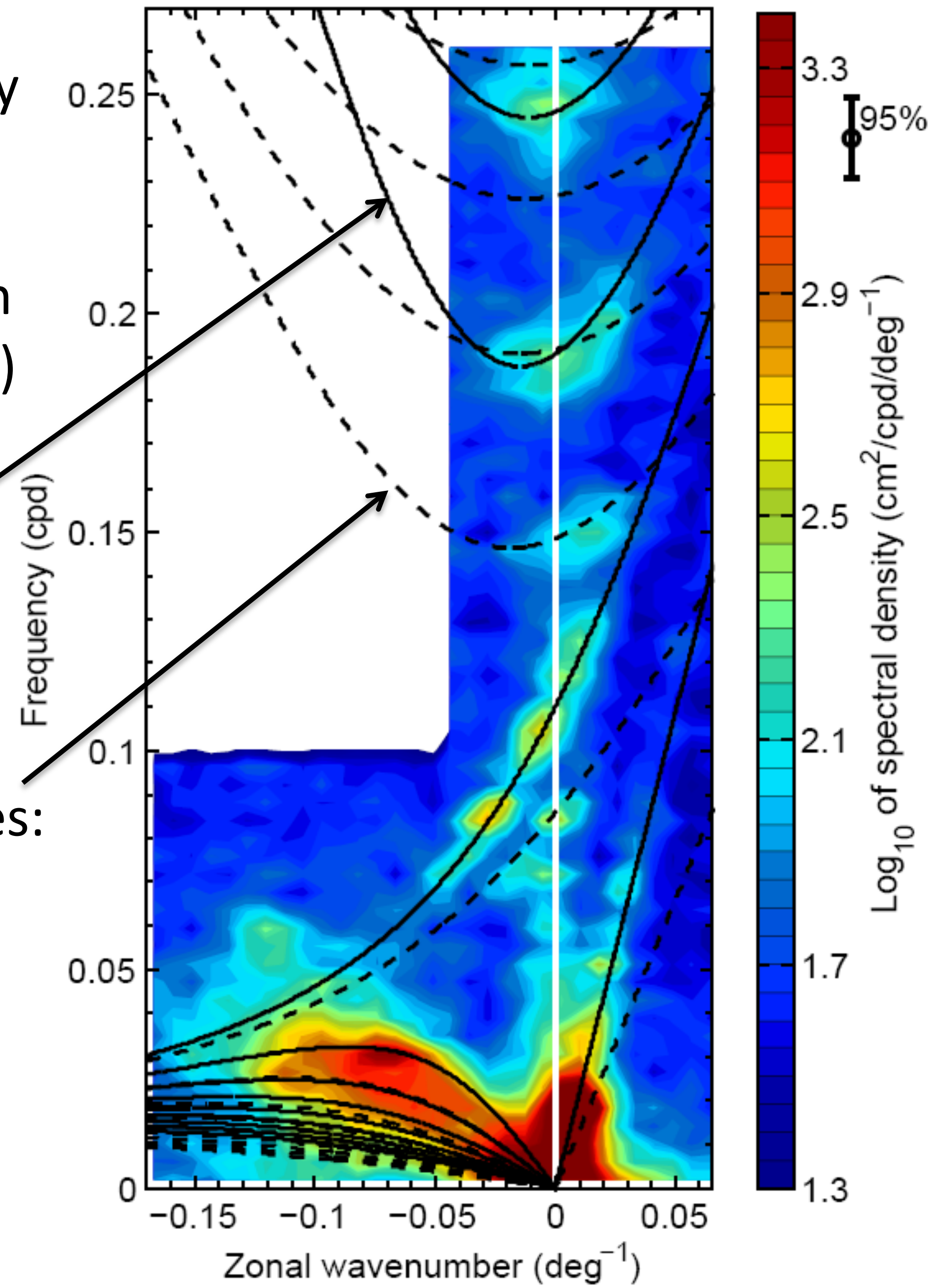
Ted Durland and Tom Farrar
May 7, 2013

Supported by NASA OVWST program

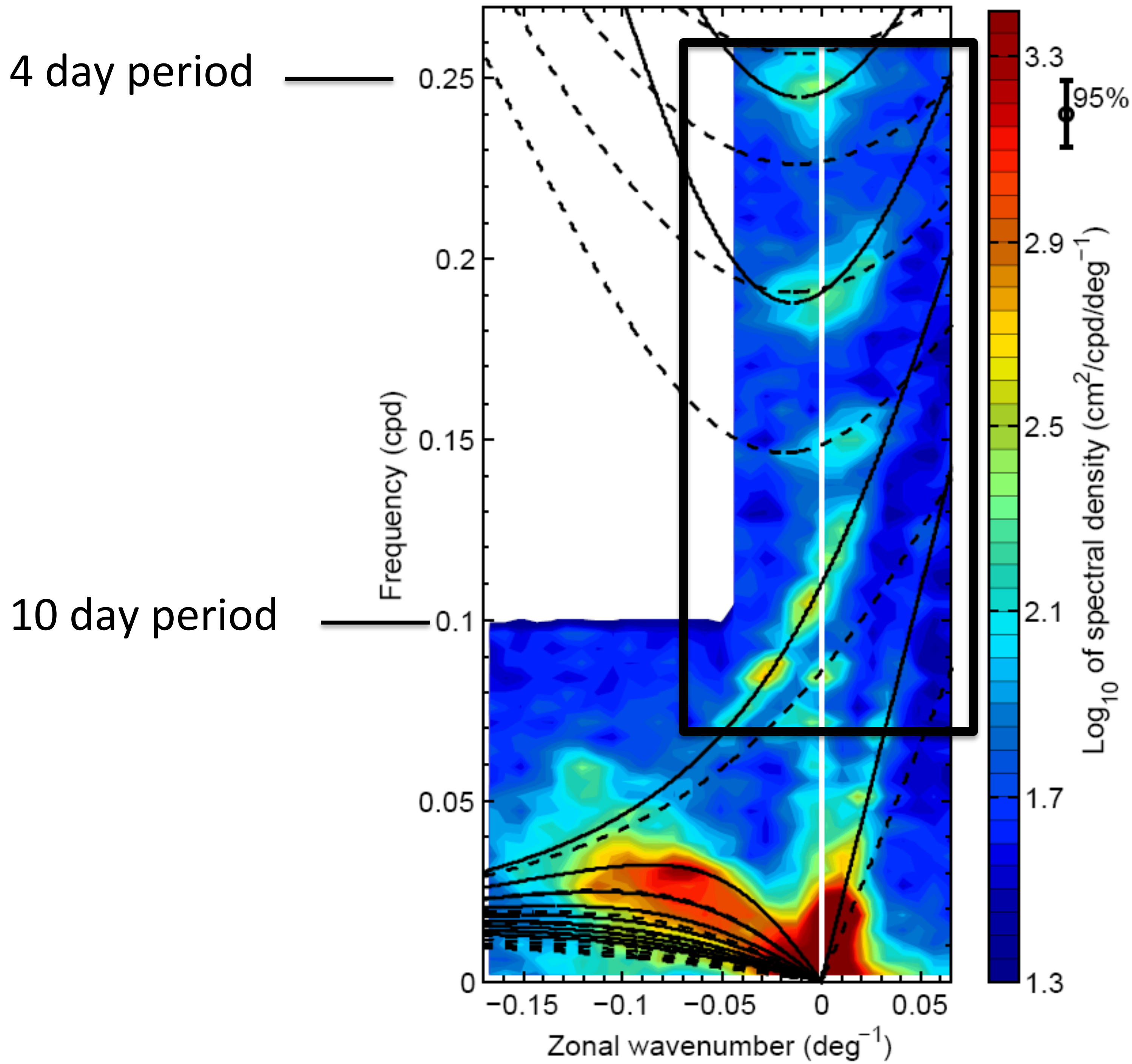
Zonal-wavenumber/ frequency spectrum of SSH from fit to along-track altimeter data in the equatorial Pacific Ocean (averaged 6°S-6°N, 2000-2005)

Solid dispersion curves: baroclinic mode 1

Dashed dispersion curves: baroclinic mode 2



(Comparable spectra of TAO-TRITON dynamic height in Farrar and Durland, *JPO*, Nov, 2012)



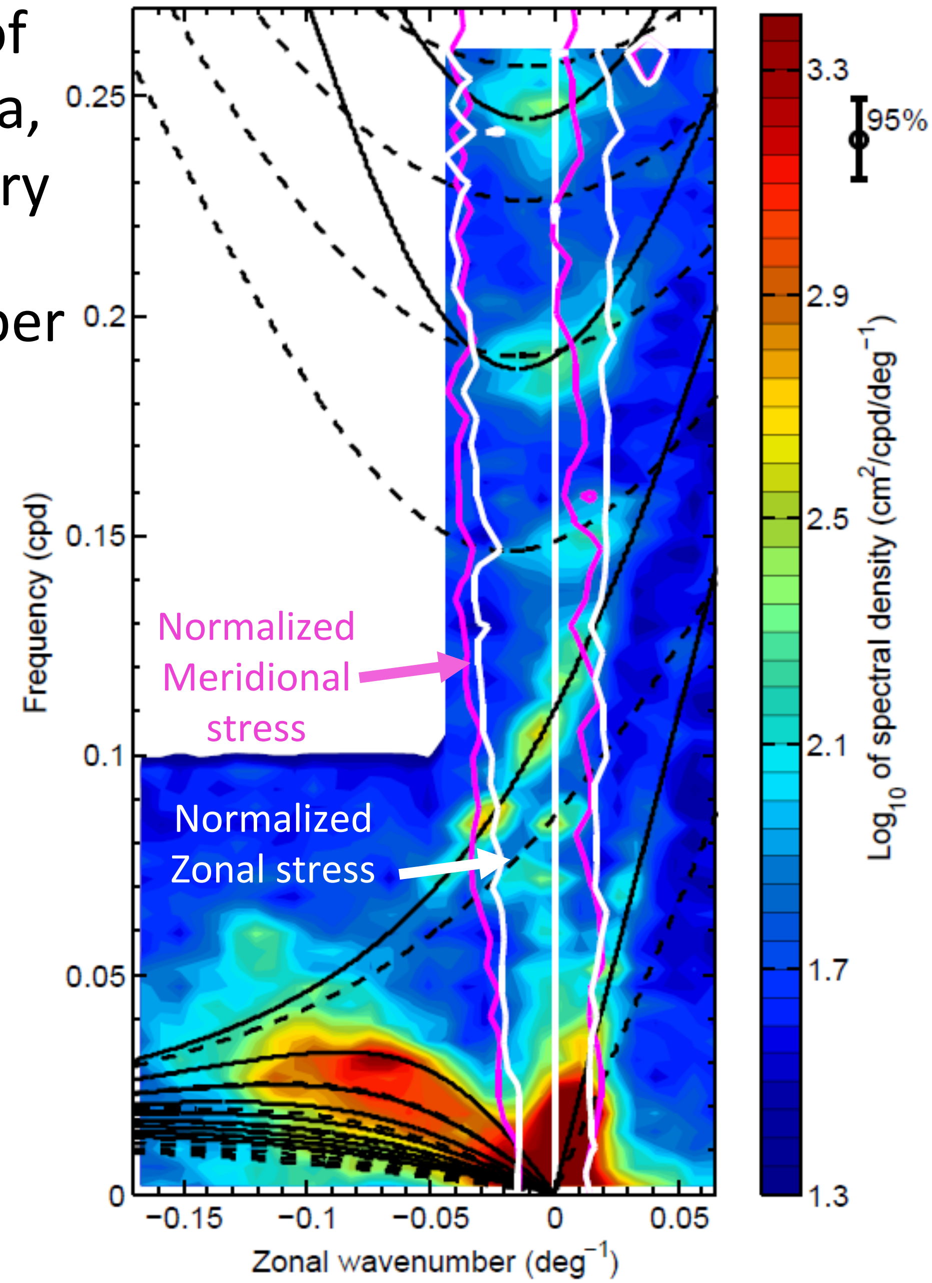
4 day period
10 day period
Frequency (cpd)
0.25
0.2
0.15
0.1
0.05
0
−0.15
−0.1
−0.05
0
0.05
Zonal wavenumber (deg^{-1})
Log$_{10}$ of spectral density (cm^2/cpd/deg^{-1})
3.3
2.9
2.5
2.1
1.7
1.3
95%

Preliminary comparisons of wind stress and SSH spectra, from QuikSCAT and altimetry

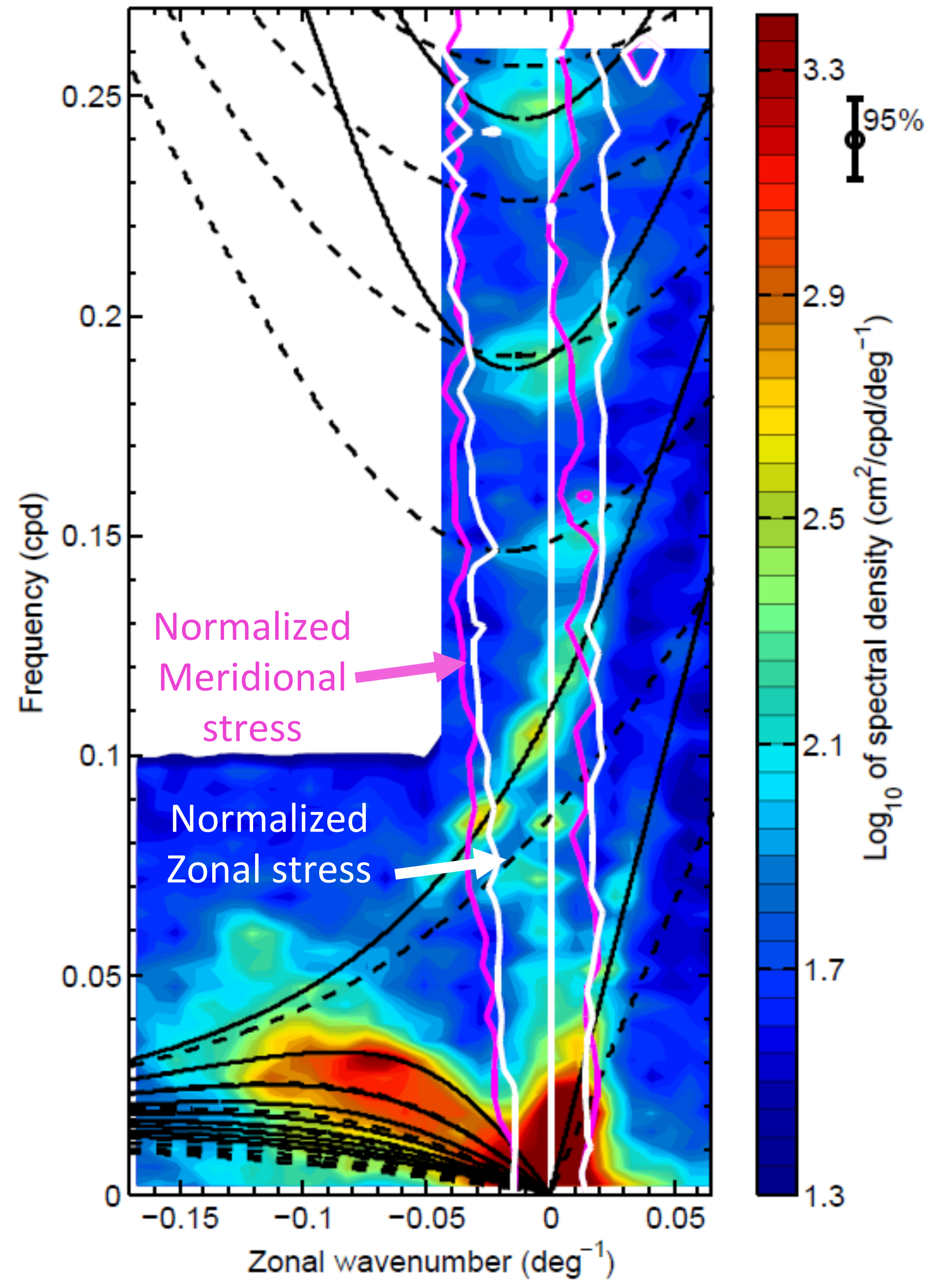
Narrow banded, low-wavenumber nature of atmospheric forcing limits oceanic resonances to low wavenumbers.

Deduced by Wunsch and Gill, 1976, from very little data.

We finally have sufficient observations to verify their conjecture.



Is the energy in these peaks reduced by the ability to leave a forcing region when the group velocity does not vanish?



Forced and damped non-dimensional equations

$$-i\sigma u - yv + ikp = 0$$

$$-i\sigma v + yu + p_y = Y_s$$

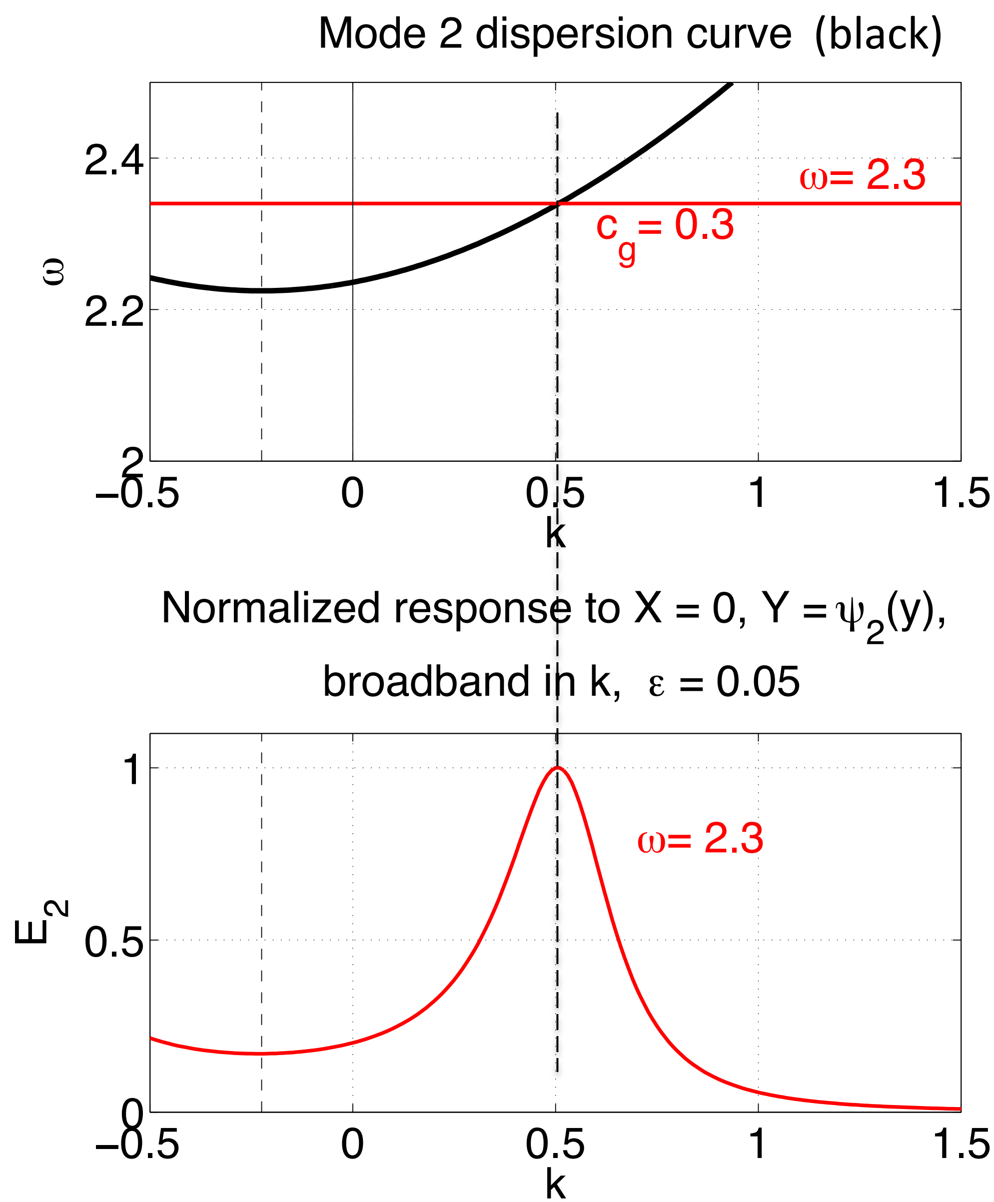
$$-i\sigma p + iku + v_y = 0$$

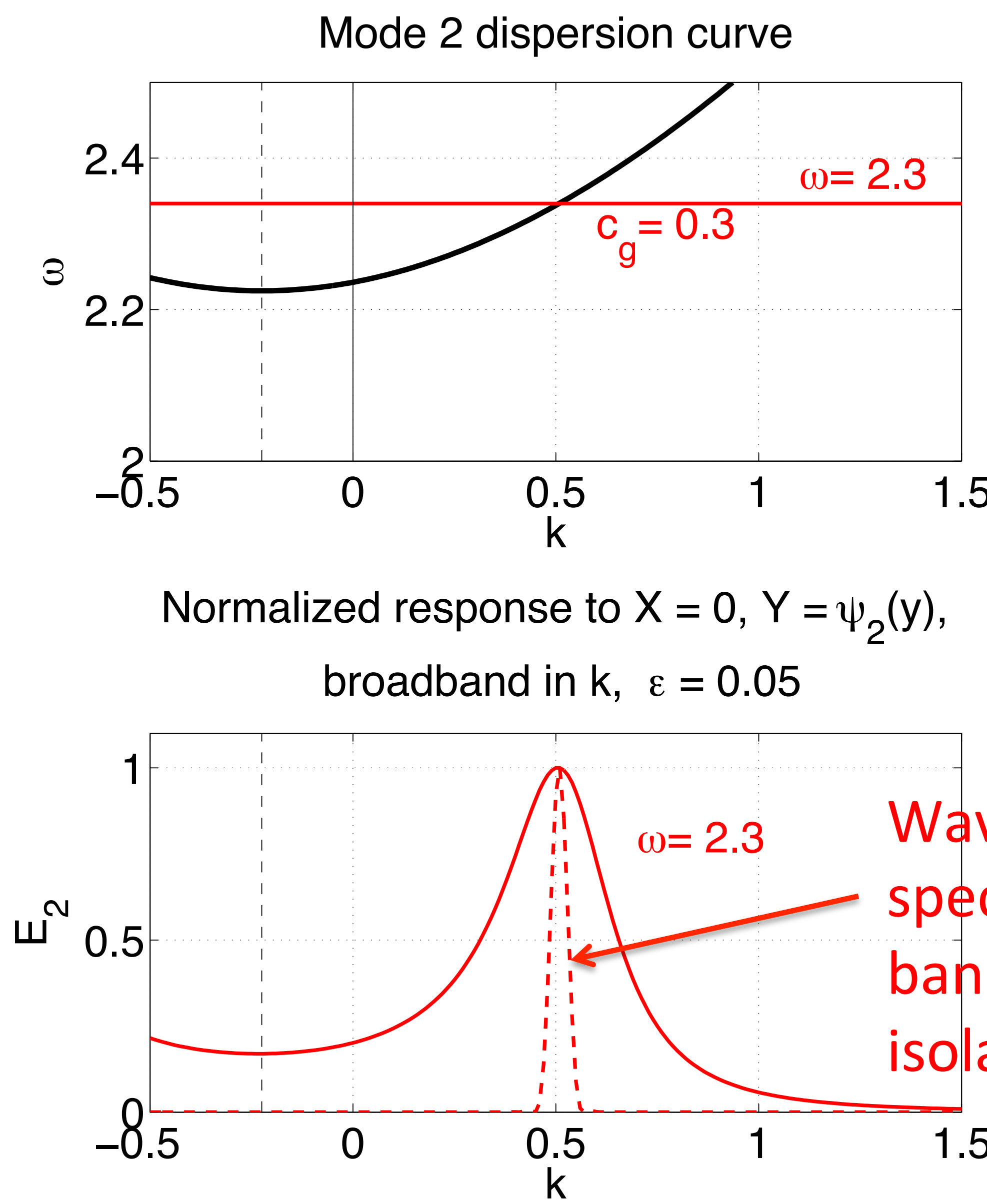
$Y_s \propto$ meridional wind stress that is symmetric about equator

$\sigma \equiv \omega + i\varepsilon$: complex frequency

ε : inverse dissipation time scale

Symmetric meridional wind stress can only force even numbered meridional modes with p (SSH) that is anti-symmetric about the equator.


Mode 2 dispersion curve (black)
ω= 2.3
c_g = 0.3
ω
2.4
2.2
2
−0.5
0
0.5
1
1.5
k
Normalized response to X = 0, Y = ψ_2(y),
broadband in k, ε = 0.05
ω= 2.3
E_2
1
0.5
0
−0.5
0
0.5
1
1.5
k



Wavenumber spectrum of narrow-banded forcing that is isolated in space

Approximations for very narrow band forcing

All forcing components in phase

$\downarrow$

All response components in phase

$$(v_{(m)}, u_{(m)}, p_{(m)}) \approx v_m \, (V_{(m)}, U_{(m)}, P_{(m)})$$

(Known meridional structures of free-wave velocity and pressure)

$$\left\langle \overline{u_{(m)} \, p_{(m)}} \right\rangle \approx E_m \, c_{gx}$$

(Energy density times zonal group velocity of free wave)

$$E_m \equiv \left| v_m \right|^2 \left\langle \overline{\left| V_{(m)} \right|^2 + \left| U_{(m)} \right|^2 + \left| P_{(m)} \right|^2} \right\rangle \propto \overline{\left\langle \left| p_{(m)} \right|^2 \right\rangle}$$

(Total wave energy density)

$\overline{\qquad\qquad}$: Period average

$\langle \quad \rangle$: Meridional integral

Simplified energy equation for meridional mode m

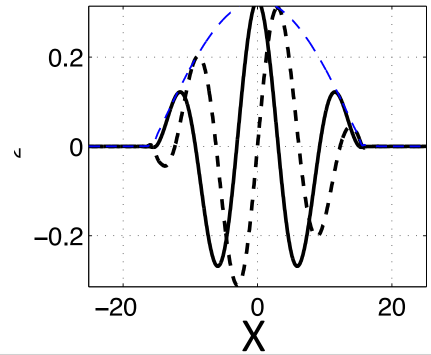
$$L_\varepsilon \frac{d}{dx} E_m + E_m = \gamma \left\langle V_{(m)}(y)\ Y_s(x,y) \right\rangle$$

where $L_\varepsilon = c_{gx} / 2\varepsilon$ is the energy dissipation length scale.

Solution:

$$E_m(x) = L_\varepsilon^{-1} \int_0^\infty \exp(-\xi / L_\varepsilon) \left\langle V_{(m)}(y)\, Y_s(x-\xi, y) \right\rangle d\xi$$

Isolated forcing at frequency where oceanic group velocity = 0.3 (nondim)

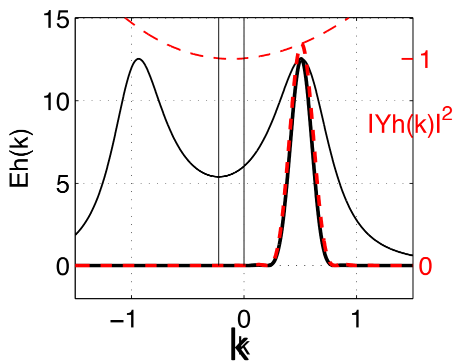


$\varepsilon = 0.10$

$L_\varepsilon = 1.5$

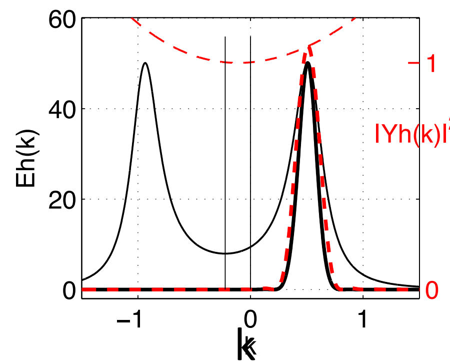
$\varepsilon = 0.05$

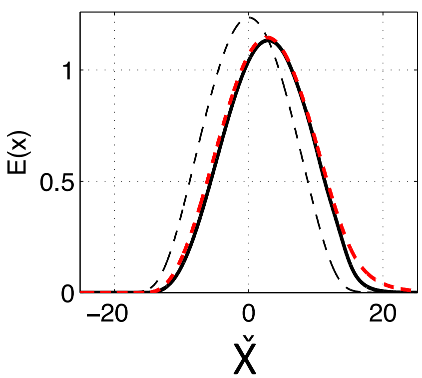
$L_\varepsilon = 3$



thin: Fourier transform of response to broadband forcing

thick: Fourier transform of narrow-band isolated forcing

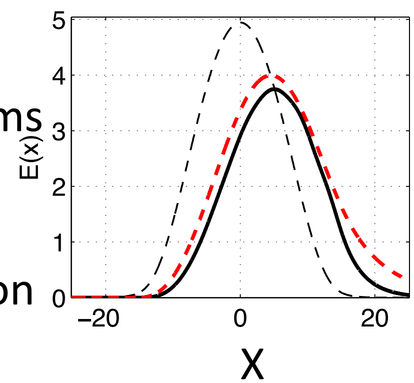




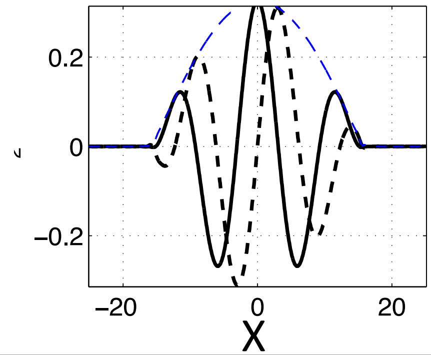
thick black: Energy solution using Fourier transforms

dashed red: 1st-order model solution

dashed black: Energy solution w/ no group velocity



Isolated forcing at frequency where oceanic group velocity = 0.3 (nondim)

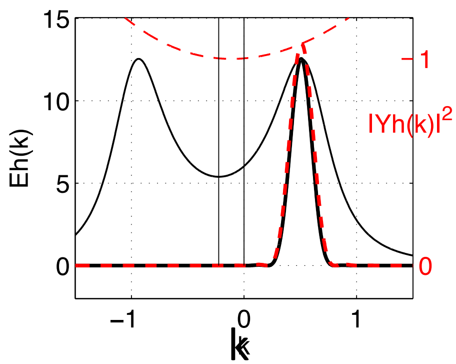


$\varepsilon = 0.10$

$L_\varepsilon = 1.5$

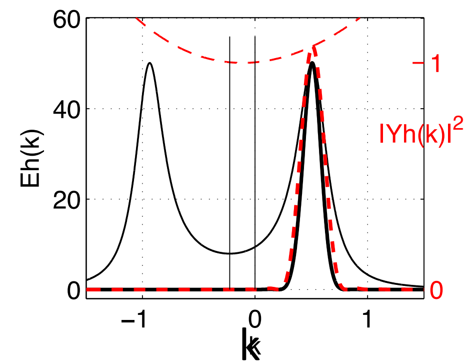
$\varepsilon = 0.05$

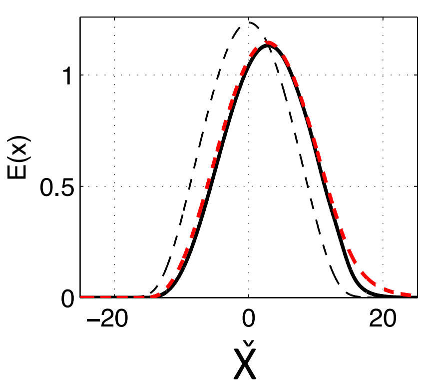
$L_\varepsilon = 3$



thin: Fourier transform of response to broadband forcing

thick: Fourier transform of narrow-band isolated forcing

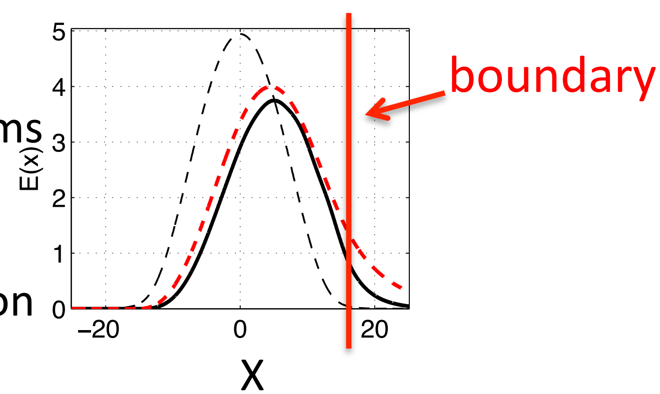




thick black: Energy solution using Fourier transforms

dashed red: 1[st]-order model solution

dashed black: Energy solution w/ no group velocity



bc mode 1, merid mode 2

4–4.1 day band,

$$c_{gx} \approx 0.15 \text{ deg / day}$$

(20 cm/s)

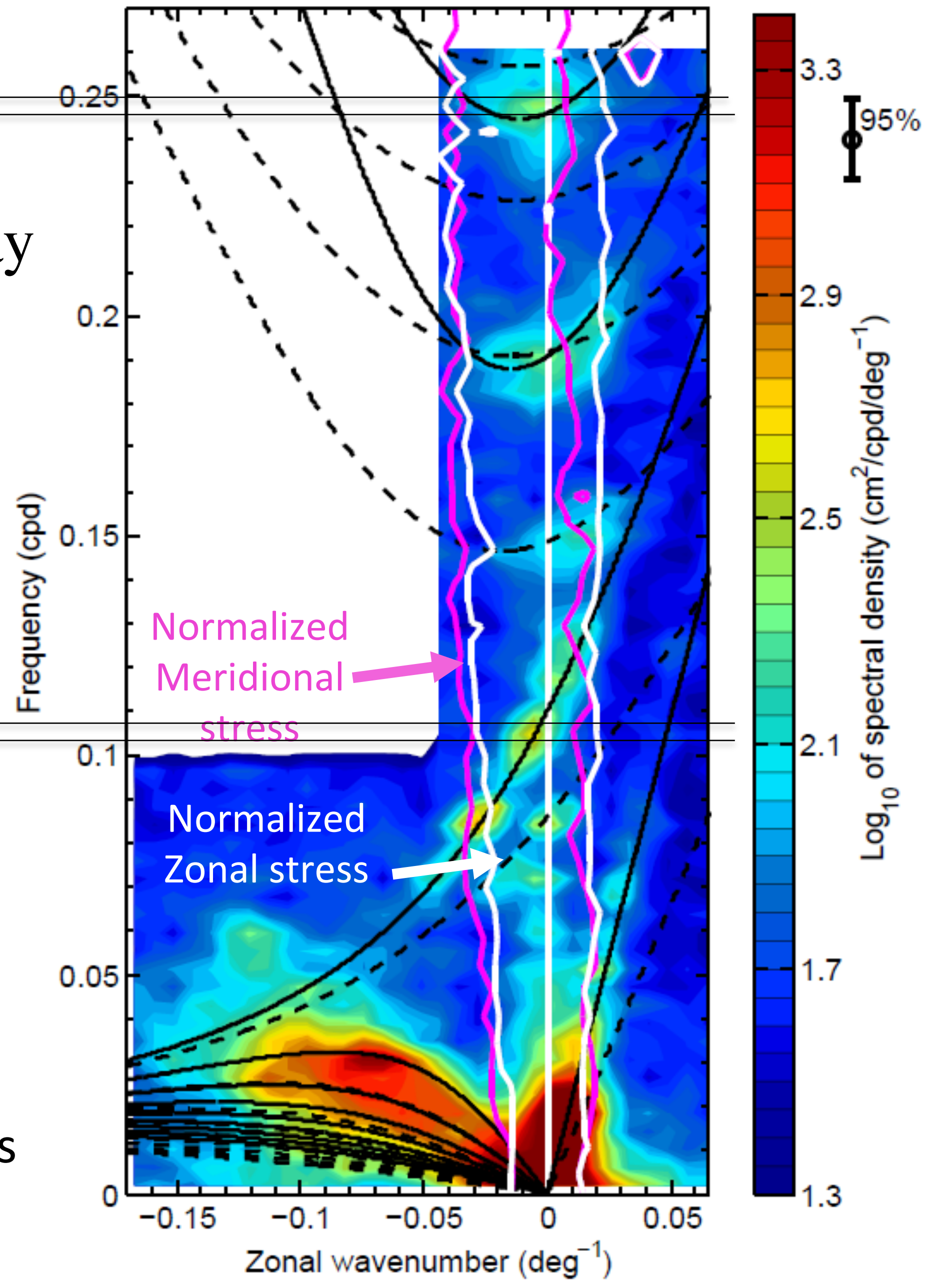
bc mode 1, merid mode 0

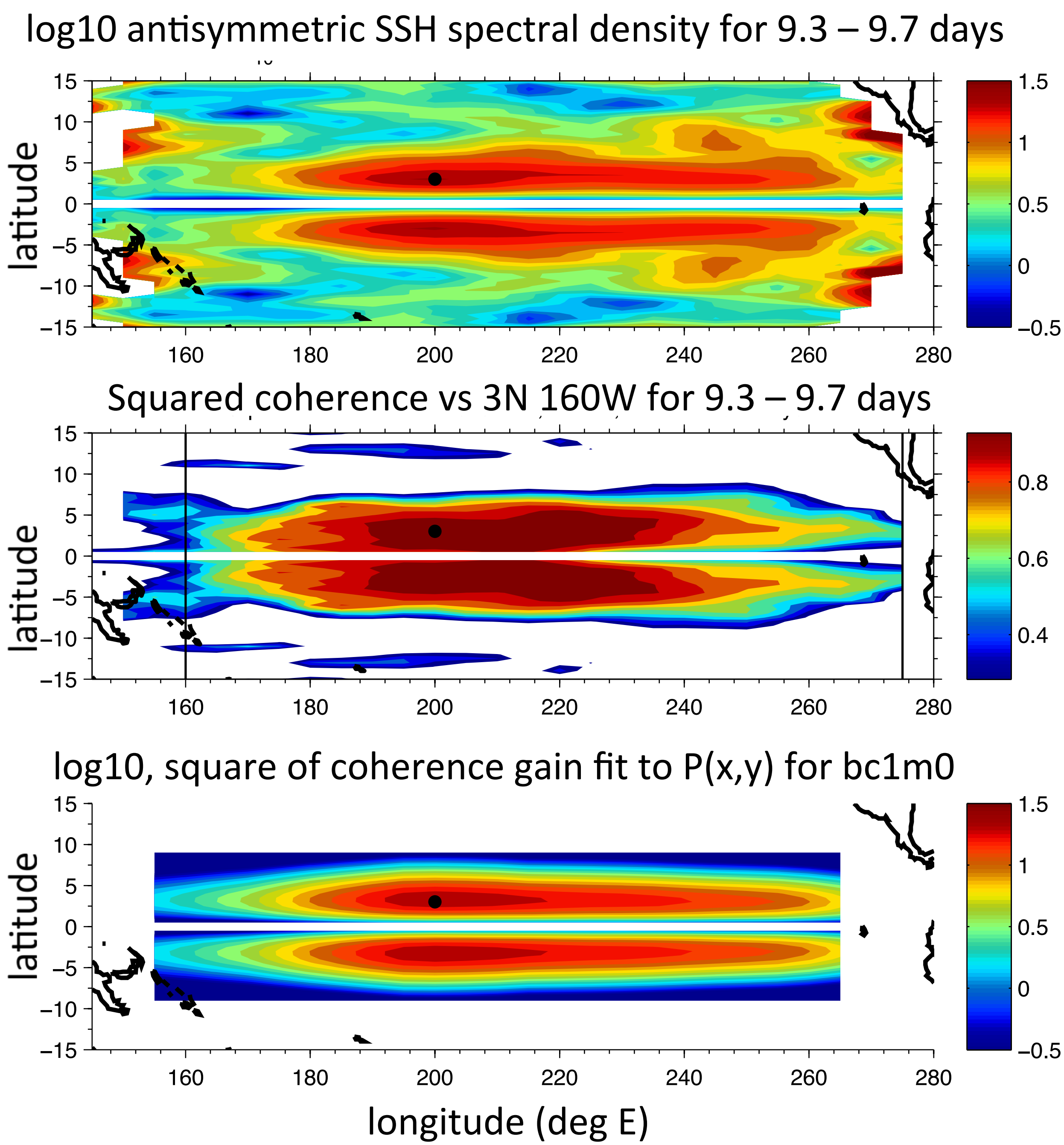
9.3–9.7 day band,

$$c_{gx} \approx 1.0 \text{ deg / day}$$

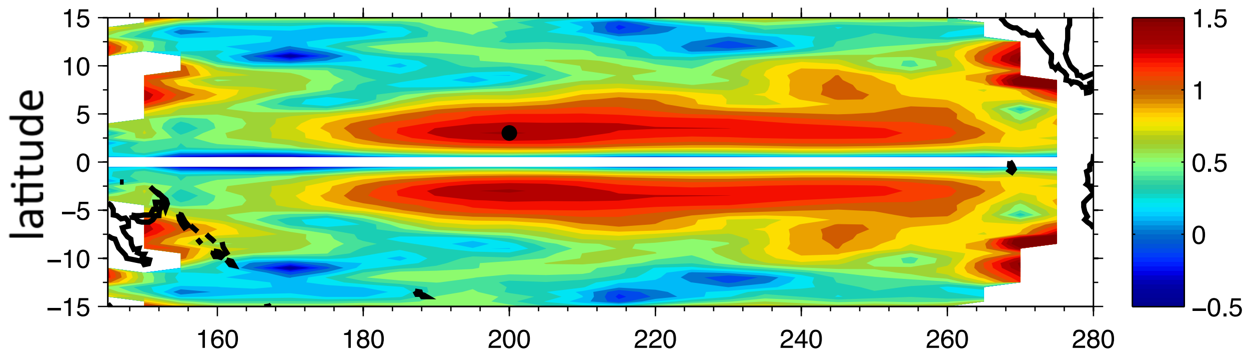
(130 cm/s)

6 year record: 10 frequency bands in each of the above bands

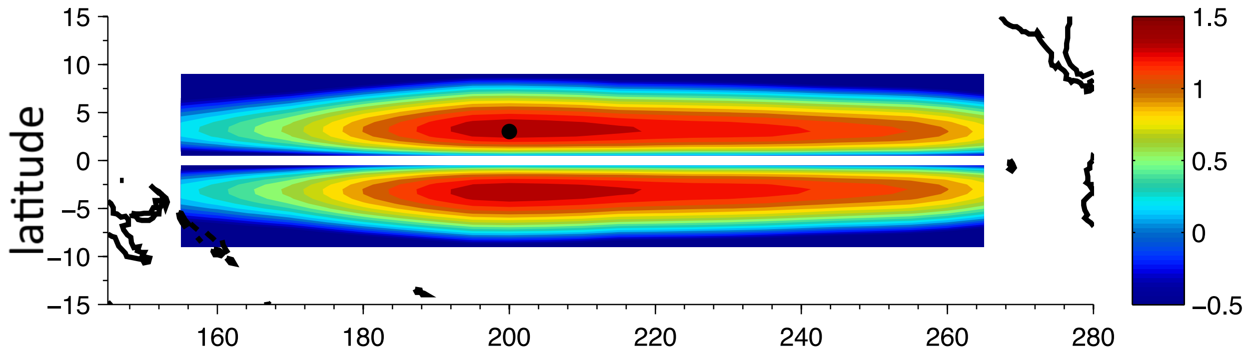



log10 antisymmetric SSH spectral density for 9.3 – 9.7 days
Squared coherence vs 3N 160W for 9.3 – 9.7 days
log10, square of coherence gain fit to P(x,y) for bc1m0
latitude
longitude (deg E)
15
10
5
0
−5
−10
−15
160
180
200
220
240
260
280
1.5
1
0.5
0
−0.5
0.8
0.6
0.4

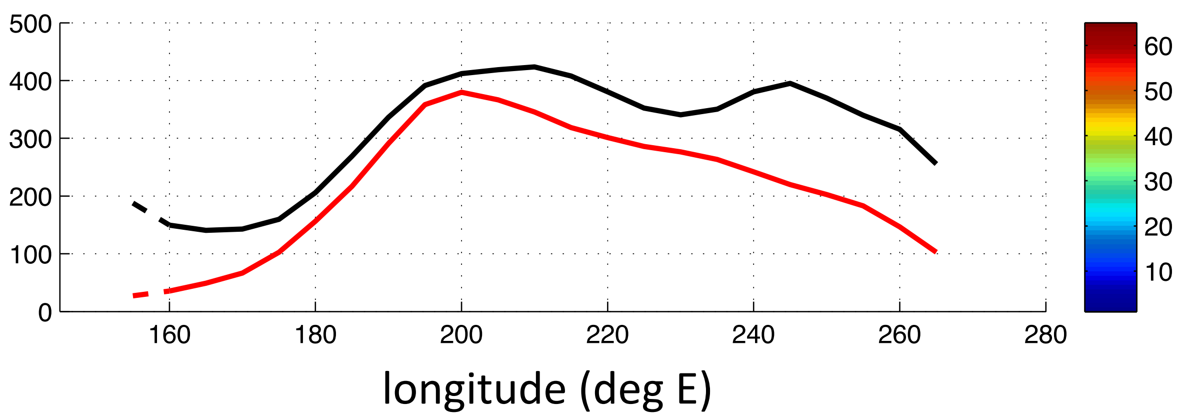
log10 antisymmetric SSH spectral density for 9.3 – 9.7 days

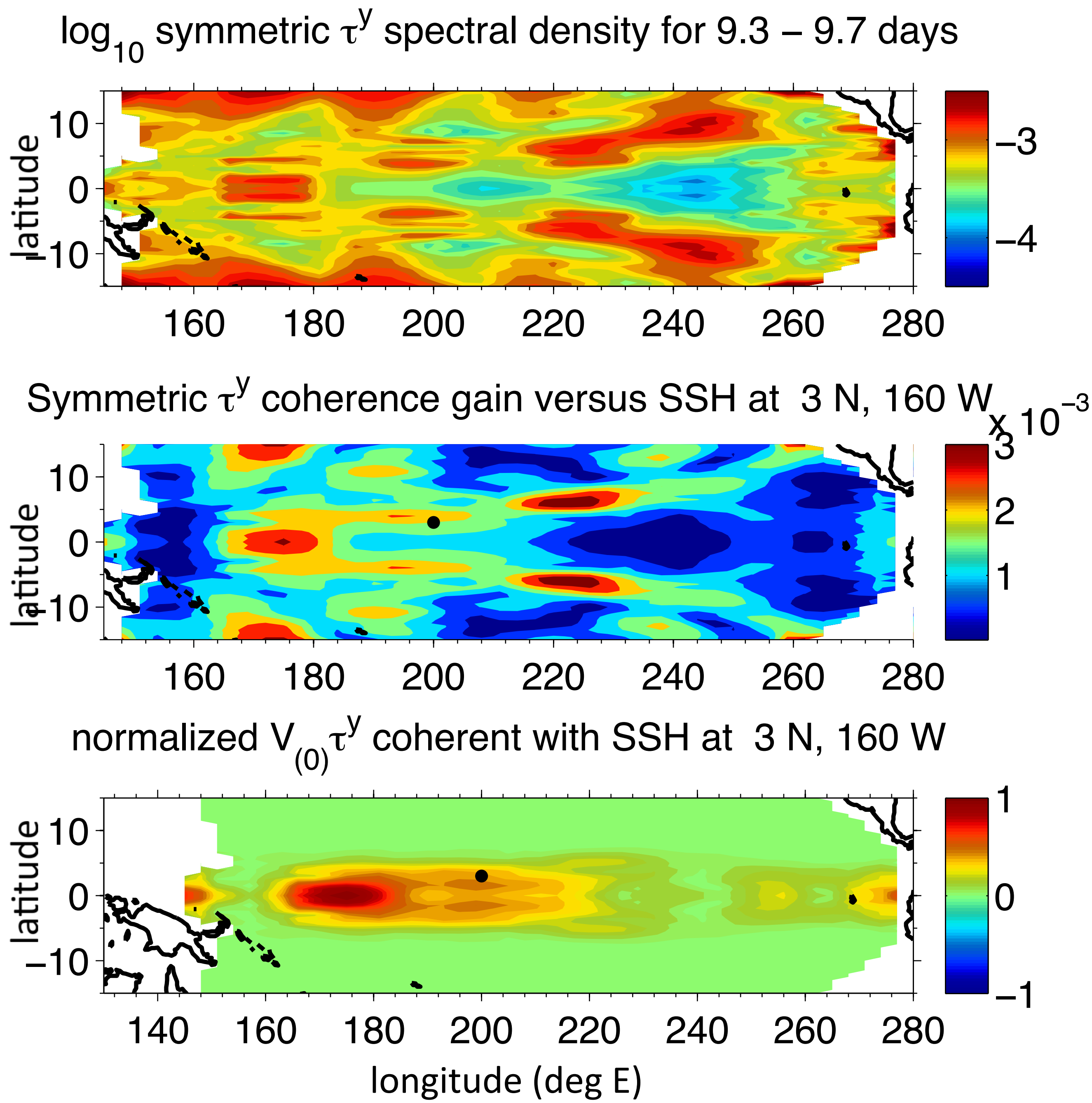


log10, square of coherence gain fit to P(x,y) for bc1m0

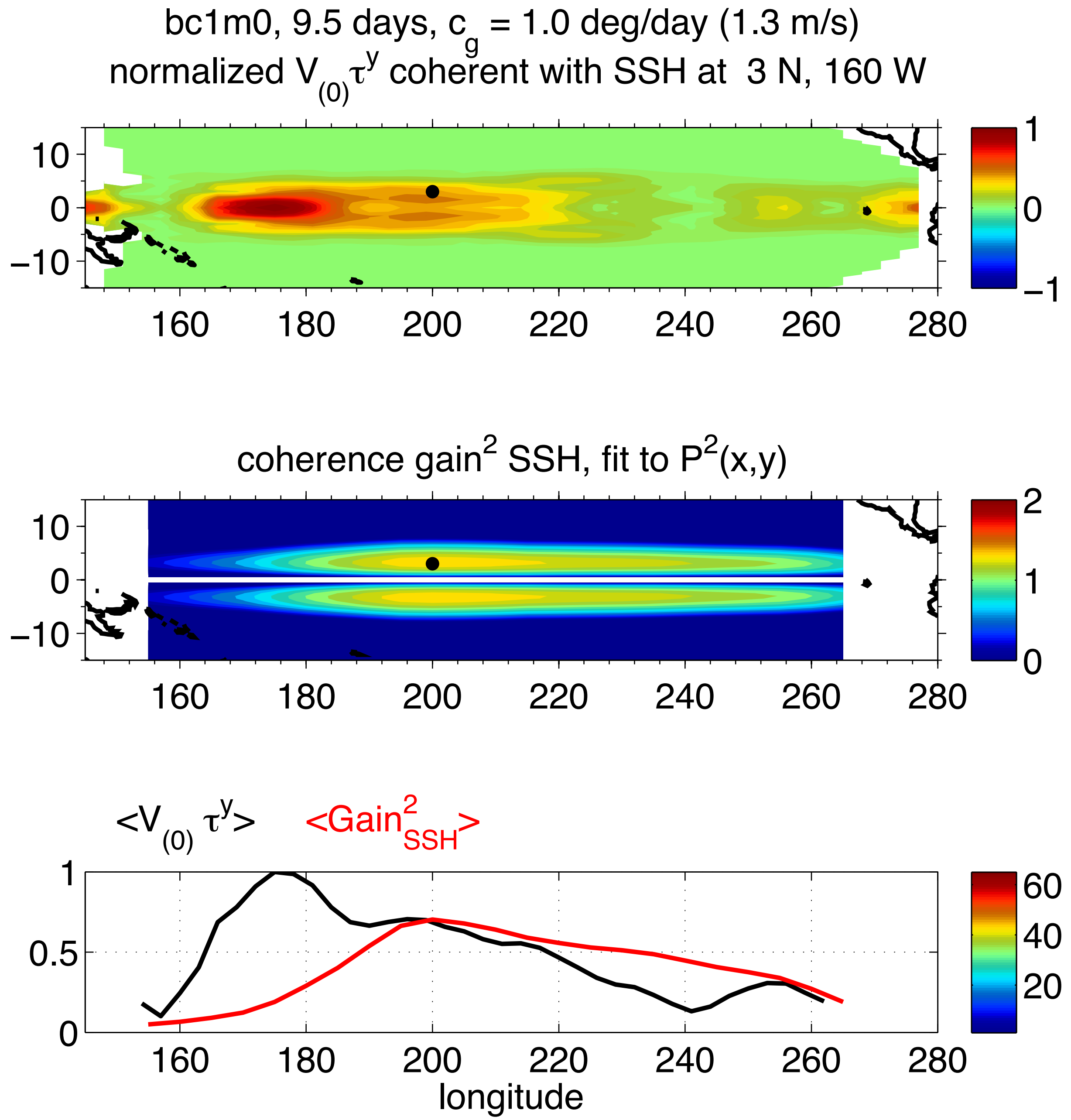


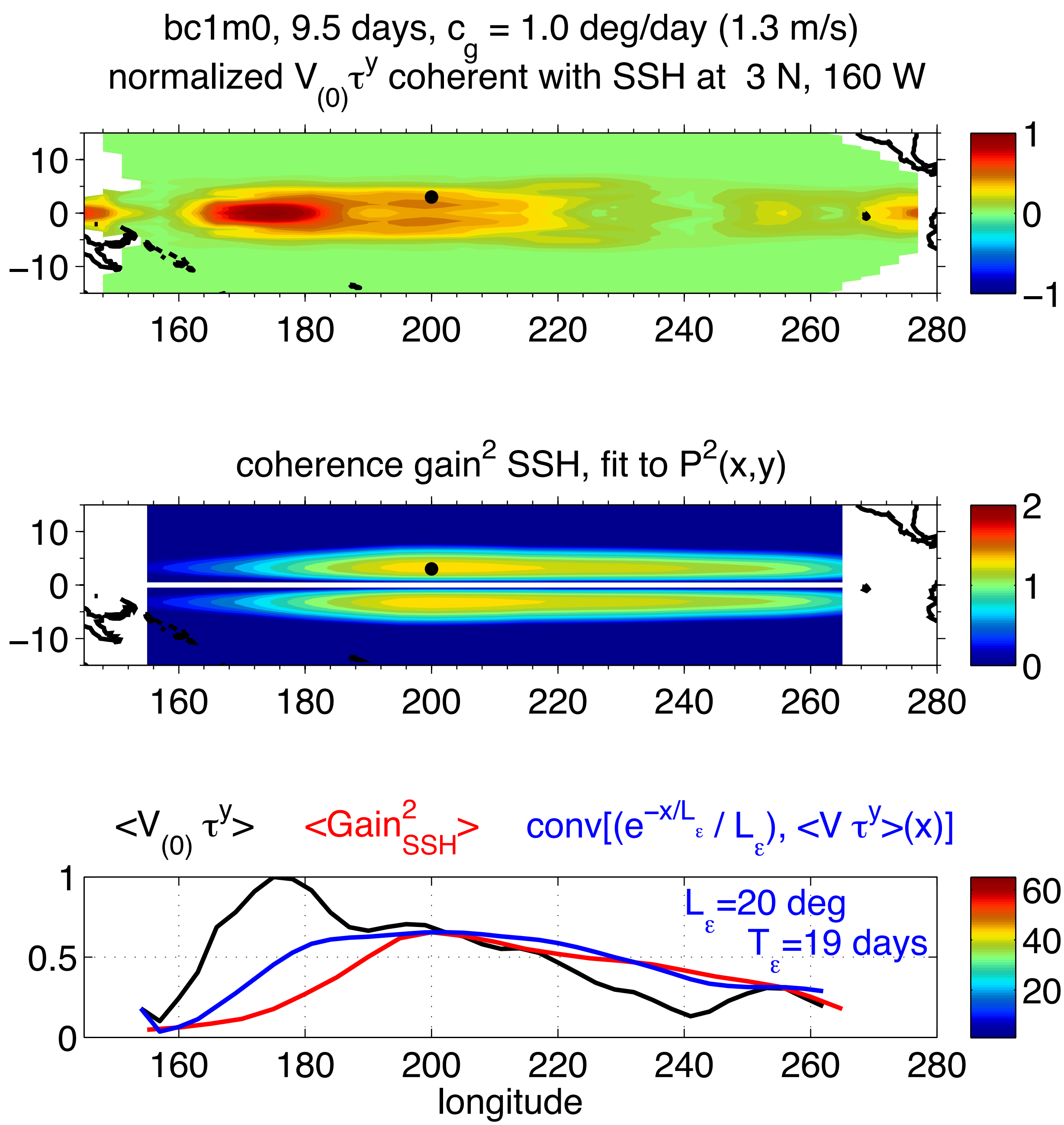
Integrated antisymmetric SSH variance. Black: total, Red: coherent bc1m0 (69%)



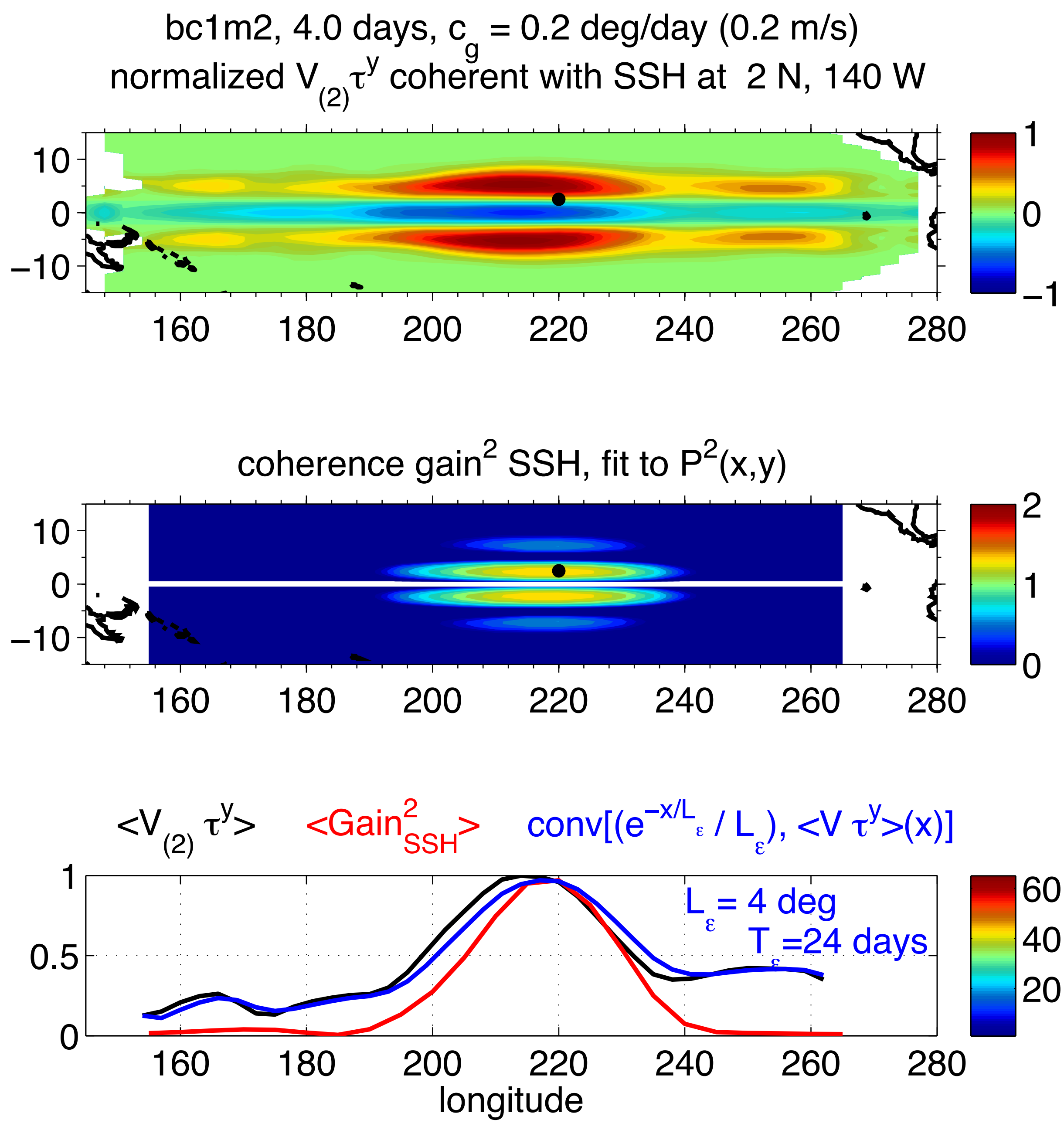


log10 symmetric τy spectral density for 9.3 – 9.7 days
latitude
10
0
−10
160
180
200
220
240
260
280
−3
−4
Symmetric τy coherence gain versus SSH at 3 N, 160 W
x 10−3
3
2
1
normalized V(0)τy coherent with SSH at 3 N, 160 W
140
1
0
−1
longitude (deg E)


bc1m0, 9.5 days, c_g = 1.0 deg/day (1.3 m/s)
normalized V_(0)τ^y coherent with SSH at 3 N, 160 W
10
0
−10
160
180
200
220
240
260
280
1
0
−1
coherence gain^2 SSH, fit to P^2(x,y)
2
1
0
<V_(0) τ^y>
<Gain^2_SSH>
0.5
20
40
60
longitude



bc1m0, 9.5 days, c_g = 1.0 deg/day (1.3 m/s)
normalized $V_{(0)}\tau^y$ coherent with SSH at 3 N, 160 W
coherence gain^2 SSH, fit to $P^2(x,y)$
$\langle V_{(0)}\ \tau^y \rangle$
$\langle Gain^2_{SSH} \rangle$
conv$[(e^{-x/L_\varepsilon} / L_\varepsilon), \langle V\ \tau^y \rangle(x)]$
L_ε =20 deg
T_ε =19 days
longitude


bc1m2, 4.0 days, c_g = 0.2 deg/day (0.2 m/s)
normalized $V_{(2)}\tau^y$ coherent with SSH at 2 N, 140 W
coherence gain^2 SSH, fit to P^2(x,y)
$<V_{(2)}\ \tau^y>$
$<Gain^2_{SSH}>$
conv[(e^{-x/L_ε} / L_ε), $<V\ \tau^y>$(x)]
L_ε = 4 deg
T_ε =24 days
longitude

Tentative Conclusions

1) Group velocity displaces oceanic response “downwind” of forcing by dissipation length scale
2) If displaced response remains within basin, energy not lost from mode
3) In the 4-10 day peaks we see, it doesn’t look like much energy is lost because of group velocity
4) Dissipation time scale can be estimated from forcing-response displacement