Evidence of and a Theory for 10-km Wavelength Convergence Lines in Synthetic Aperture Radar Tropical Cyclone Surface Wind Retrievals

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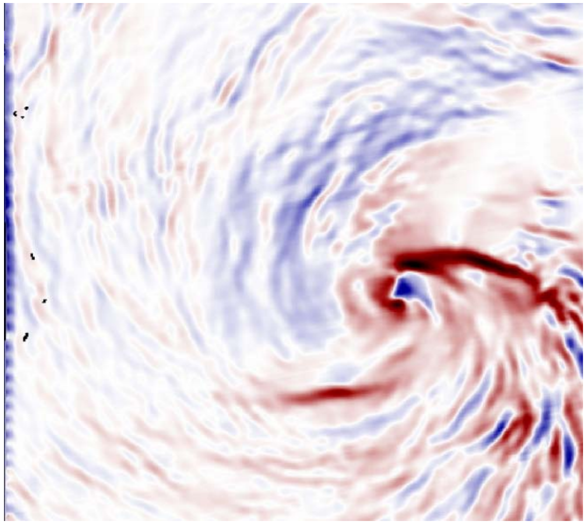
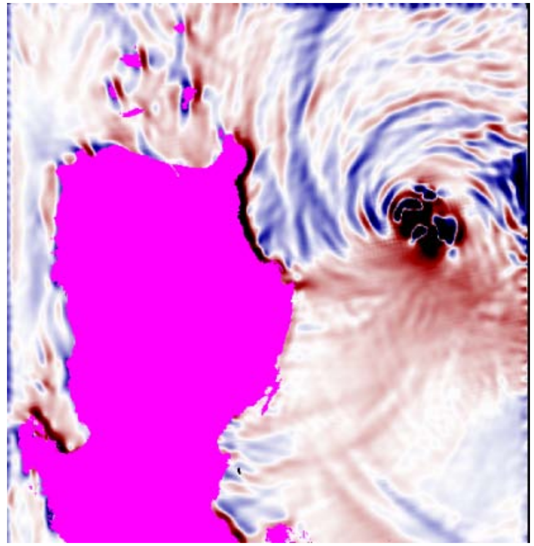
Research supported by NASA IOVWST and ONR Physical Oceanography & Marine Meteorology

Multi-km-Scale Surface Wind Conv./Div. Patterns

Red: Conv.
Blue: Div.

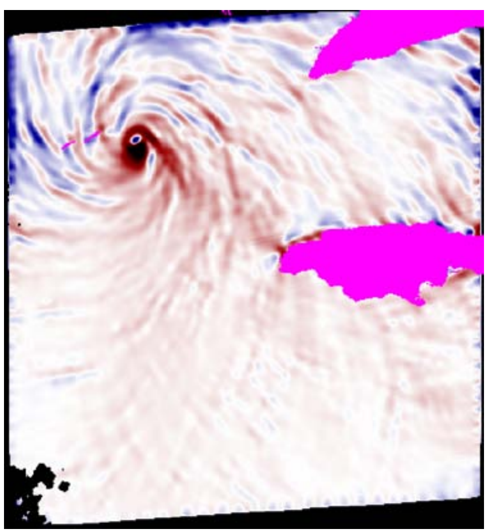


Malakas 22

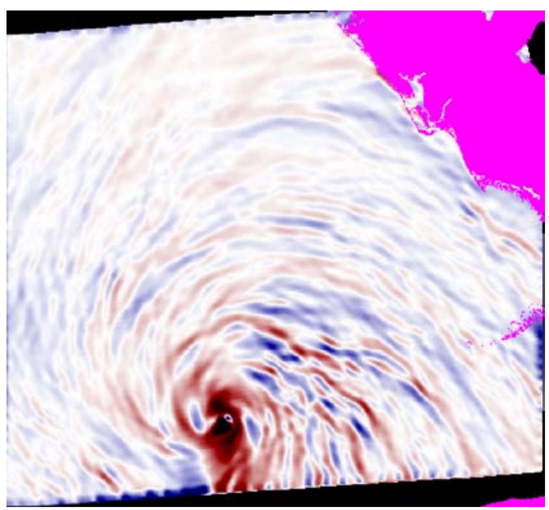


Megi 17

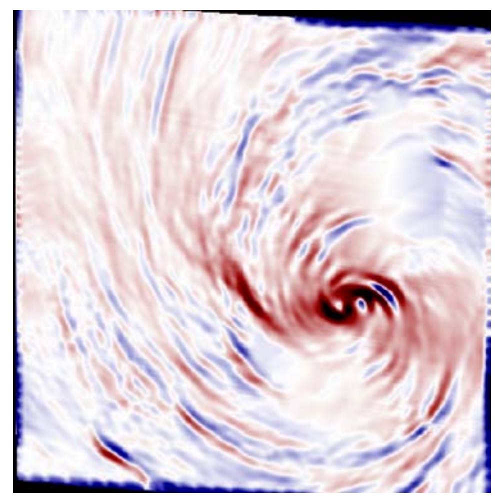
Ike



Lili



Katrina 27



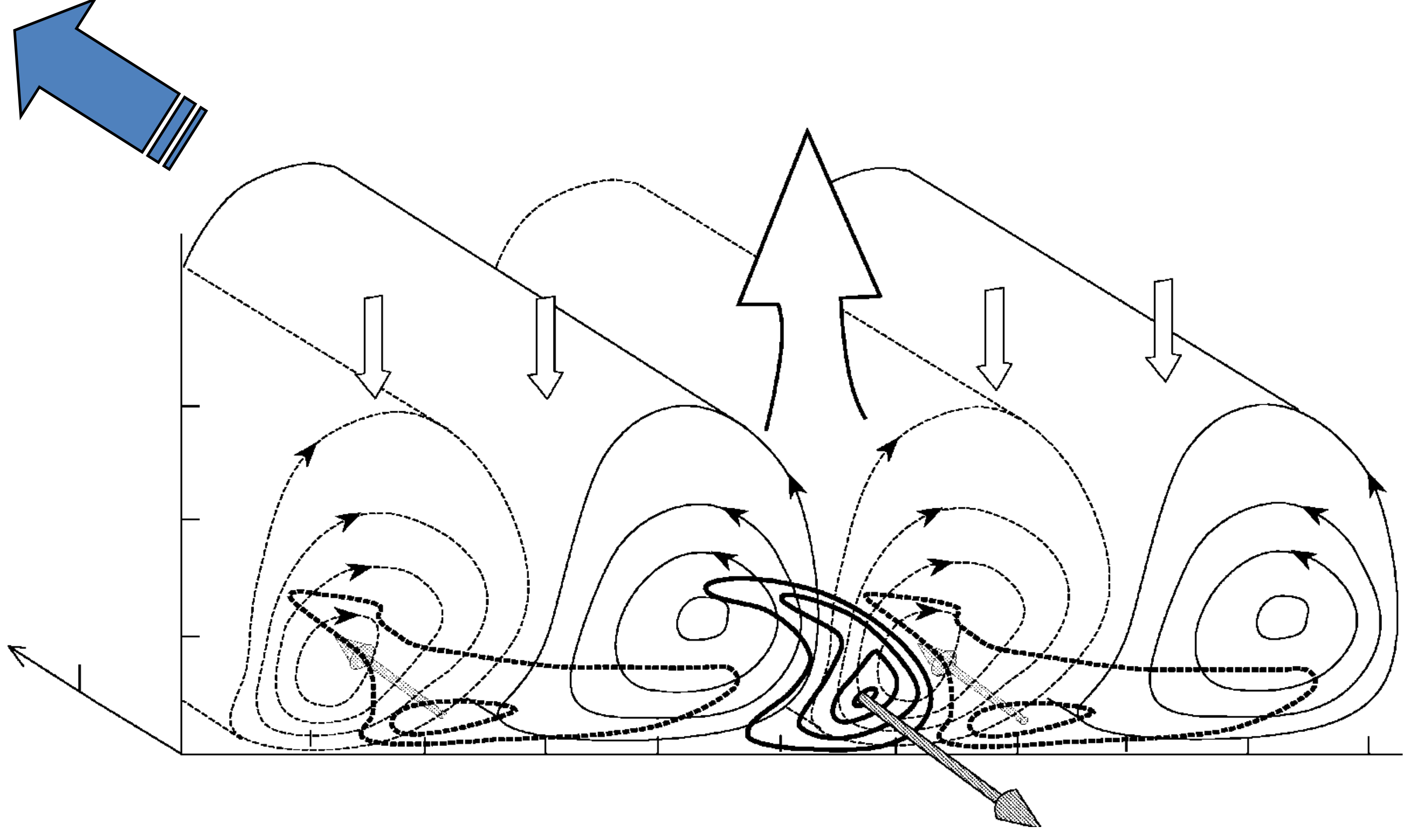
Helene 20

SLP-Filtered Tropical Cyclone SAR Wind Vector Fields (see poster)

- SLP acts as a low-pass filter
  - Forces dynamical consistency
  - Winds at 1 km pixels
- Can generate sensible derivative fields
- Consistent signature in all SAR typhoon/hurricane divergence fields
  - *O*(~10 km) wavelength convergence bands
  - Look like PBL rolls, but aspect ratio (λ/h) is too large

Science Hypothesis

- The convergence lines are the signature of large (O(10+)) aspect ratio (λ/h) TC PBL roll vortices
  - Theory and observations agree that most common TC PBL rolls have aspect ratio O(2-4) (*fastest-growing*)
  - *Very Slowly-growing* large aspect ratio rolls are present, but are not expected to survive competition with faster growing dominant rolls.
- Proposed Mechanism: Upscale transfer of energy from dominant modes into weak modes through low-order resonant triad wave-wave interaction
  - 8^{th}-order theory based on 2-D Ekman layer model of Mourad and Brown (1990); plus 6 contributions omitted in M&B



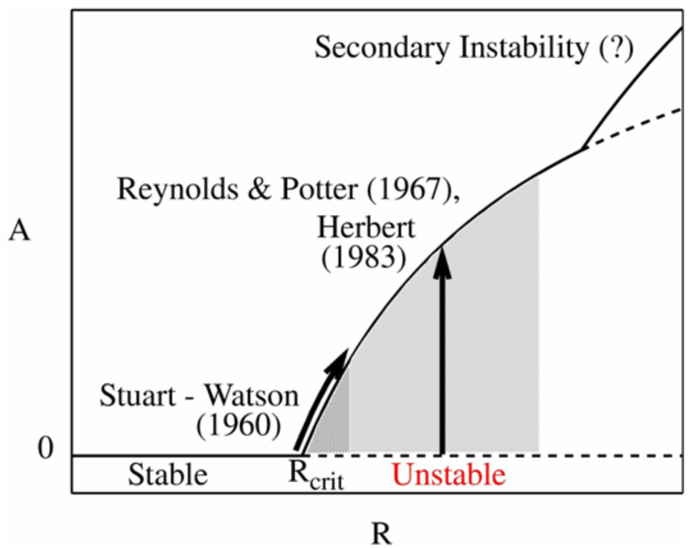
Wavelength: Larger-scale structures ~ 1500 to 2000 m
Smaller-scale structures ~ 300 to 700 m

Velocity Perturbations: +/- 7 m s^{-1} typical
up to +/- "10s of" m s^{-1} small-scale

Orientation: Typically along-mean TCBL wind, wide variability

Prevalence: Roll-scale structures ~ unknown, (35% to 70%)
Streak-scale structures: ***Most likely usually present***

Single-Wave Roll Theory
Nonlinear Stability



$$\lambda = a + i\omega = \frac{1}{A}\frac{dA}{dt} + i\frac{d\eta}{dt} = \lambda_0 + A^2\lambda_1 + A^4\lambda_2 + \cdots$$

$$\underset{\sim}{q} = 2real\left[\sum_{n=0}^{\infty} A^n e^{in(\alpha x - \omega t)} \sum_{m=0}^{\infty} A^{2m} \underset{\sim}{q}_{nm}(z)\right]$$

- "Stretch" eigenvalue, λ_0, in powers of nonlinear amplitude, *A(t)*.
- Expand eigenfunction, q_{10}, in harmonics of fundamental wavenumber, α and forced modifications
  - Forced fundamental modifications are orthogonal to linear mode
  - Determine Landau Coefficients (the λ_i)
- Find equilibrium solution ($dA/dt = 0$).

Standard Single-Wave PBL Roll Model

Table 5.1 Contributions to the nonlinear perturbation up to the fifth Landau Coefficient

Order	Landau	q_0	q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9	q_{10}	q_{11}
1		MF											
A	λ_0		q_{10}										
A^2		q_{01}		q_{20}									
A^3	λ_1		q_{11}		q_{30}								
A^4		q_{02}		q_{21}		q_{40}							
A^5	λ_2		q_{12}		q_{31}		q_{50}						
A^6		q_{03}		q_{22}		q_{41}		q_{60}					
A^7	λ_3		q_{13}		q_{32}		q_{51}		q_{70}				
A^8		q_{04}		q_{23}		q_{42}		q_{61}		q_{80}			
A^9	λ_4		q_{14}		q_{33}		q_{52}		q_{71}		q_{90}		
A^{10}		q_{05}		q_{24}		q_{43}		q_{62}		q_{81}		q_{100}	
A^{11}	λ_5		q_{15}		q_{34}		q_{53}		q_{72}		q_{91}		q_{110}

Truncated Contributions to Multi-Wave Roll Model

$\mathbf{q} = [u, v, w, T]^T$

2^{nd}-order in each component
→ 8^{th}-order system

To 1^{st} Nonlinear Landau Term:

$0 \quad + A^2 q_{01}$ (mean flow modification)

$Aq_{10} + 0 \quad + A^3 q_{11}$ (fundamental wavelength)

$0 \quad + A^2 q_{20}$ (1^{st} harmonic)

$0 \quad + 0 \quad + A^3 q_{30}$ (2^{nd} harmonic)

Upscale Transfer Resonant Triad

- $\alpha = \beta + \gamma$ (mode A, mode B, mode C)
- My nonlinear solution method restricts me[1] to all unstable modes

[1]but not necessarily reality

- Require at least one wavenumber at fastest growing mode
- For upscale transfer (into small wavenumber), intermediate wavenumber is usually also fast-growing

Truncated 3-Wave Roll Solutions

$$q_\alpha = A q_{0,\alpha} + \boxed{BC q_{1,\alpha} e^{i\phi}} + A[A^2 q_{2,\alpha} + B^2 q_{3,\alpha} + C^2 q_{4,\alpha}] + A^2 q_{20,\alpha} + A^3 q_{30,\alpha}$$

$$q_\beta = B q_{0,\beta} + \boxed{AC q_{1,\beta} e^{-i\phi}} + B[A^2 q_{2,\beta} + B^2 q_{3,\beta} + C^2 q_{4,\beta}] + B^2 q_{20,\beta} + B^3 q_{30,\beta}$$

$$q_\gamma = C q_{0,\gamma} + \boxed{AB q_{1,\gamma} e^{-i\phi}} + C[A^2 q_{2,\gamma} + B^2 q_{3,\gamma} + C^2 q_{4,\gamma}] + C^2 q_{20,\gamma} + C^3 q_{30,\gamma}$$

$$\phi = \theta_A - \theta_B - \theta_C$$

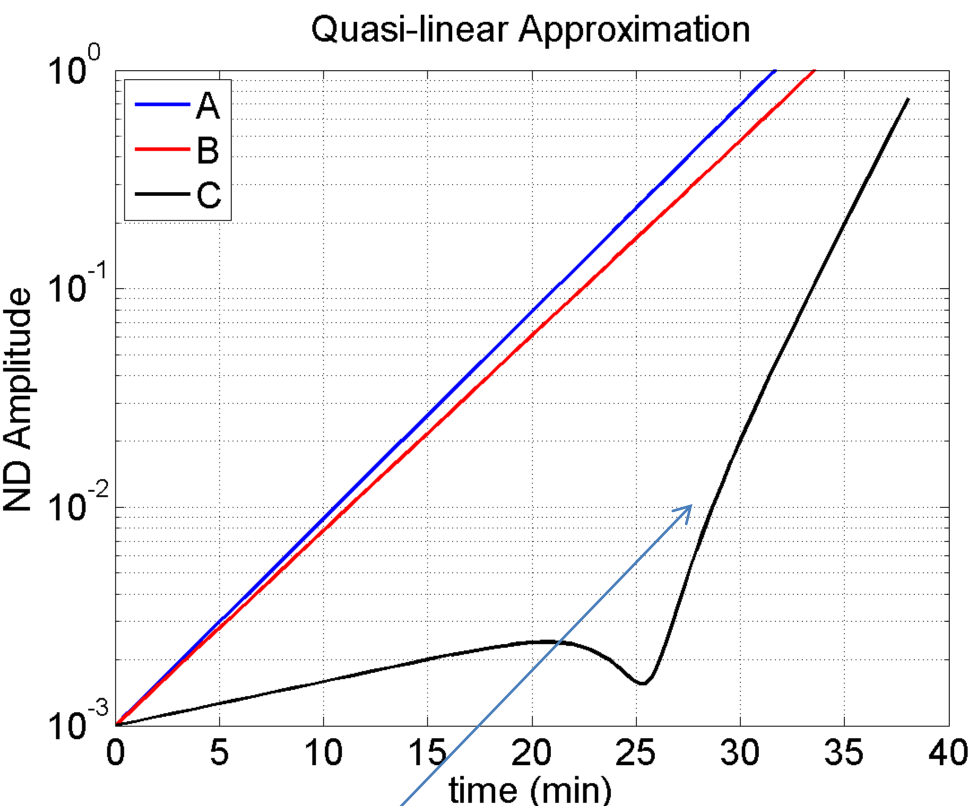
- **YELLOW**: contributions from single-wave theory. $q_{2,\alpha} = q_{11,\alpha}$, generates Landau coefficient
- **BLUE**: new wave-wave & wave-mean flow interaction terms. Each of these generates a new Landau coefficient.
- **RED:** Low-order phase-coupling wave-wave transfer terms
- Also: mean-flow modifications due to each wave

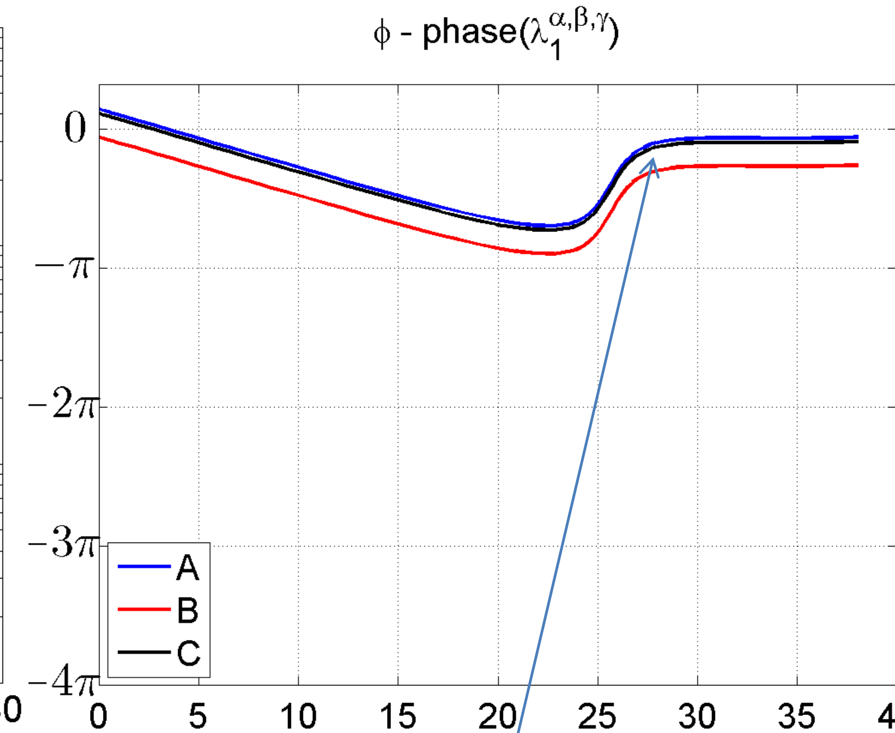
Truncated 3-Wave Model
Amplitude (real) and Phase (imaginary)

- $\frac{1}{A}\frac{dA}{dt} - i\,\frac{d\theta_A}{dt} = a_0 + \boxed{a_1 \frac{BC}{A} e^{i\phi}} + [a_2 A^2 + a_3 B^2 + a_4 C^2]$
- $\frac{1}{B}\frac{dB}{dt} - i\,\frac{d\theta_B}{dt} = b_0 + \boxed{b_1 \frac{AC}{B} e^{-i\phi}} + [b_2 A^2 + b_3 B^2 + b_4 C^2]$
- $\frac{1}{C}\frac{dC}{dt} - i\,\frac{d\theta_C}{dt} = c_0 + \boxed{c_1 \frac{AB}{C} e^{-i\phi}} + [c_2 A^2 + c_3 B^2 + c_4 C^2]$
- $\phi = \theta_A - \theta_B - \theta_C$ (Wave phase imbalance)
- $\alpha = \beta + \gamma$ (resonant triad wavenumbers)

  - The a_i, b_i, c_i are Landau coefficients, calculated via an orthogonalization assumption (nonlinear wave-wave & wave-mean flow interactions)
  - <u>Highest-order</u> (bracketed) terms <u>force equilibrium</u>; dominated by single-wave contributions (a_2A^2, b_3B^2, c_4C^2)
  - <u>Lower-order</u> phase coupling allows inter-scale energy transfer, **<u>always</u>** drives energy into slowest-growing mode before exponential growth is checked

Phase-Coupling:
Energizing Slowest-Growing Mode

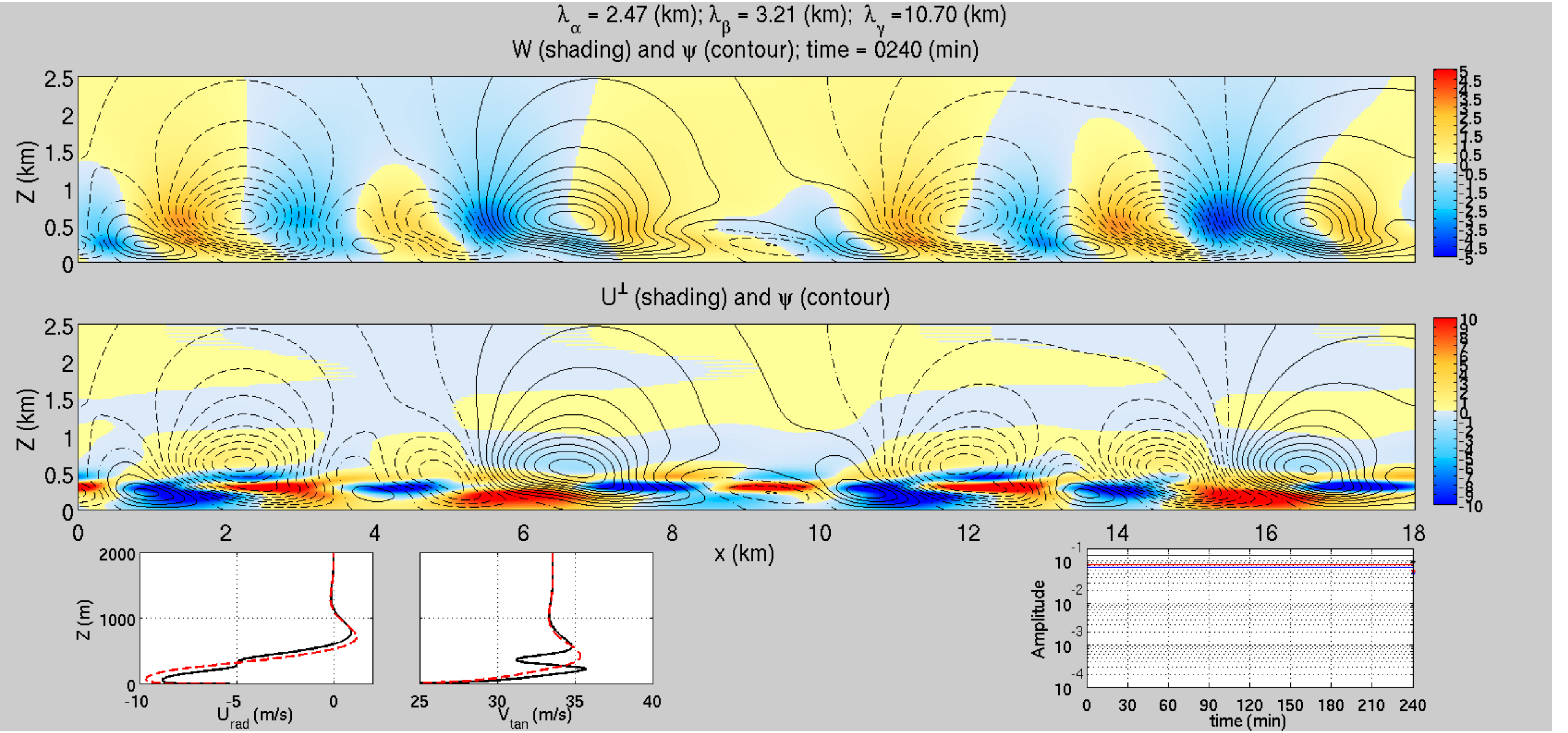




Accelerated growth of slow mode (C)

Starts when (ϕ - phase(c_1)) → $2\pi n$
This always happens in the quasi-linear stage!

$$\phi = \theta_A - \theta_B + \theta_C$$



Contours are Overturning Flow Streamfunction

Colors are: top, vertical velocity (W) (top); bottom, along-roll ($U^{\perp}$)

Note: Large aspect ratio modes extends above the PBL

Any Other Evidence?

Small-Scale Spiral Bands Observed in Hurricanes Andrew, Hugo, and Erin

MWR, **126**, 1998

ROBERT GALL, JOHN TUTTLE, AND PETER HILDEBRAND

National Center for Atmospheric Research, Boulder, Colorado

(Manuscript received 7 July 1997, in final form 10 October 1997)

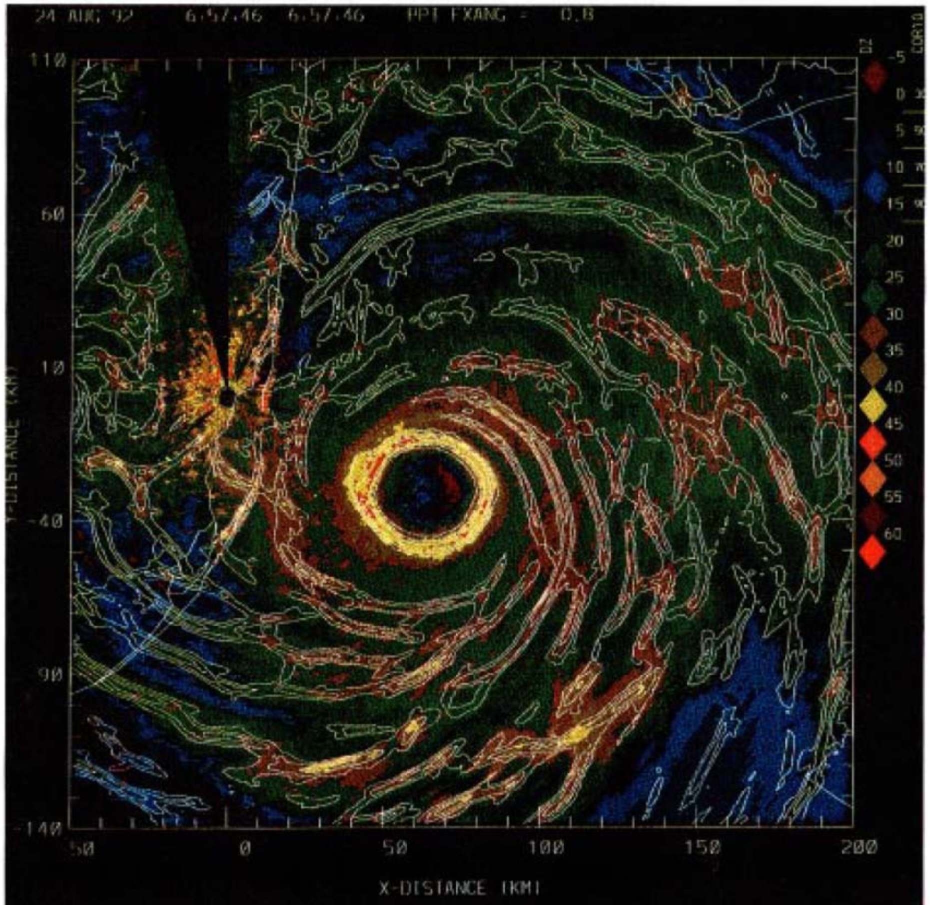


FIG. 5. Correlation field from the correlation analysis of the Andrew data from Fig. 1, using $\lambda = 10$, white contours, superimposed on the Fig. 1 radar reflectivity field. Contours start at 0.3 and increment by 0.1. Reflectivity scale same as Fig. 1.

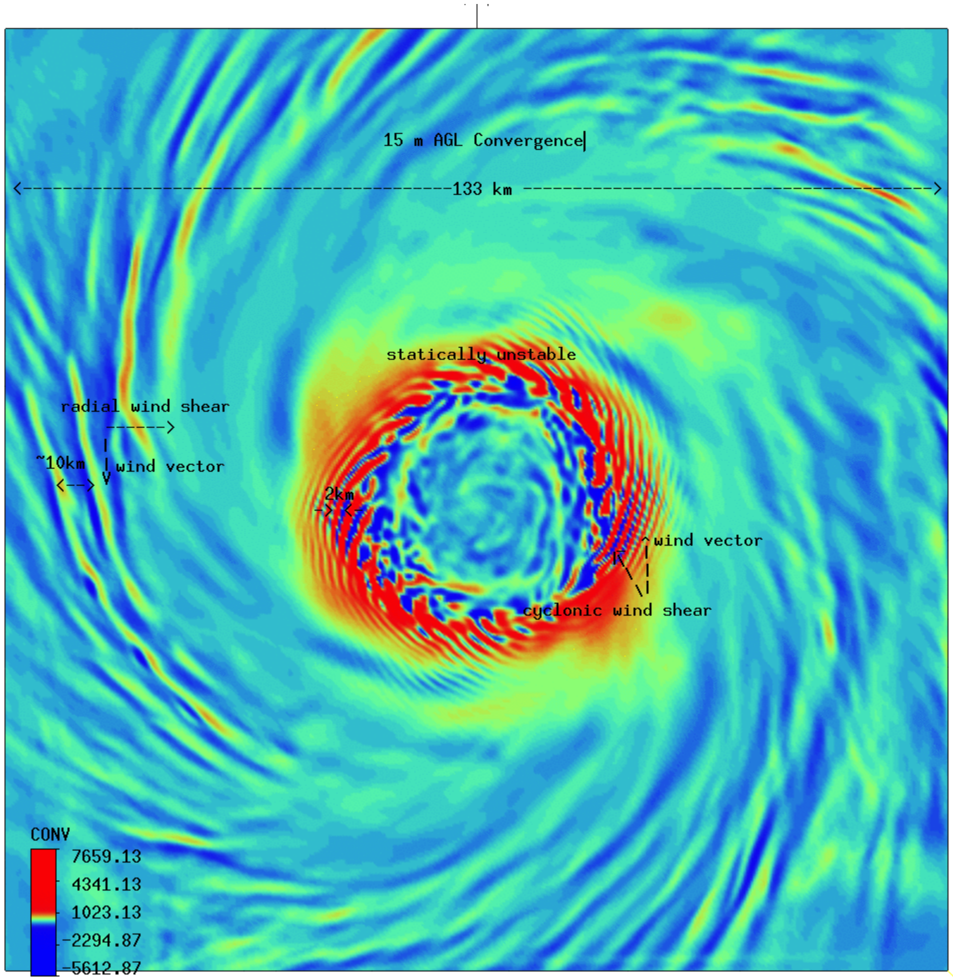
Note: Their definition of aspect ratio is different

5. Conclusions

The above analysis provides clear evidence for the existence of relatively small-scale (~10 km) spiral features of rather deep vertical extent near the eyes of three hurricanes. We suspect that these small-scale spiral features are present in most intense hurricanes. Using the radar and in situ data, we have been able to describe many features of these small-scale spiral bands, but the question remains, what are they? At this point we can only offer suggestions, since a definitive description of their structure is not available from the data at hand.

We know that the small-scale bands are on the order of 5 km deep and have spatial scales on the order of 10 km. The higher-reflectivity regions have stronger updrafts and greater equivalent potential temperature than do regions of lower reflectivity. These small-scale bands appear to line up approximately with the low-level wind, where the radial component of the flow is part of the total wind. They move approximately with the mean tangential wind throughout their depth. They exist near the center of the hurricane where the environment is saturated nearly everywhere and the lapse rates are neutral to moist ascent. It is our suggestion that these small-scale rainbands are similar to the rolls that exist in the planetary boundary layer and other flows, that are driven by the boundary layer shear in the presence of convection. Such rolls often have 1:2 aspect ratios, such as described here. We are confused by the depth of the these hurricane bands, however, since they are much deeper than we would have expected for classic boundary layer rolls. Perhaps the environment in the inner 100 km or so of the hurricane is unstable enough, wet enough, and contains enough liquid water (so that weak downdrafts associated with the band structure undergo moist adiabatic processes) and these properties hold through deep enough layers that deep structures typical of boundary layer rolls could develop. Definitive an-

Zach Gruskin (grad student) and Prof. Greg Tripoli, Univ. Wisconsin (pers. comm.)



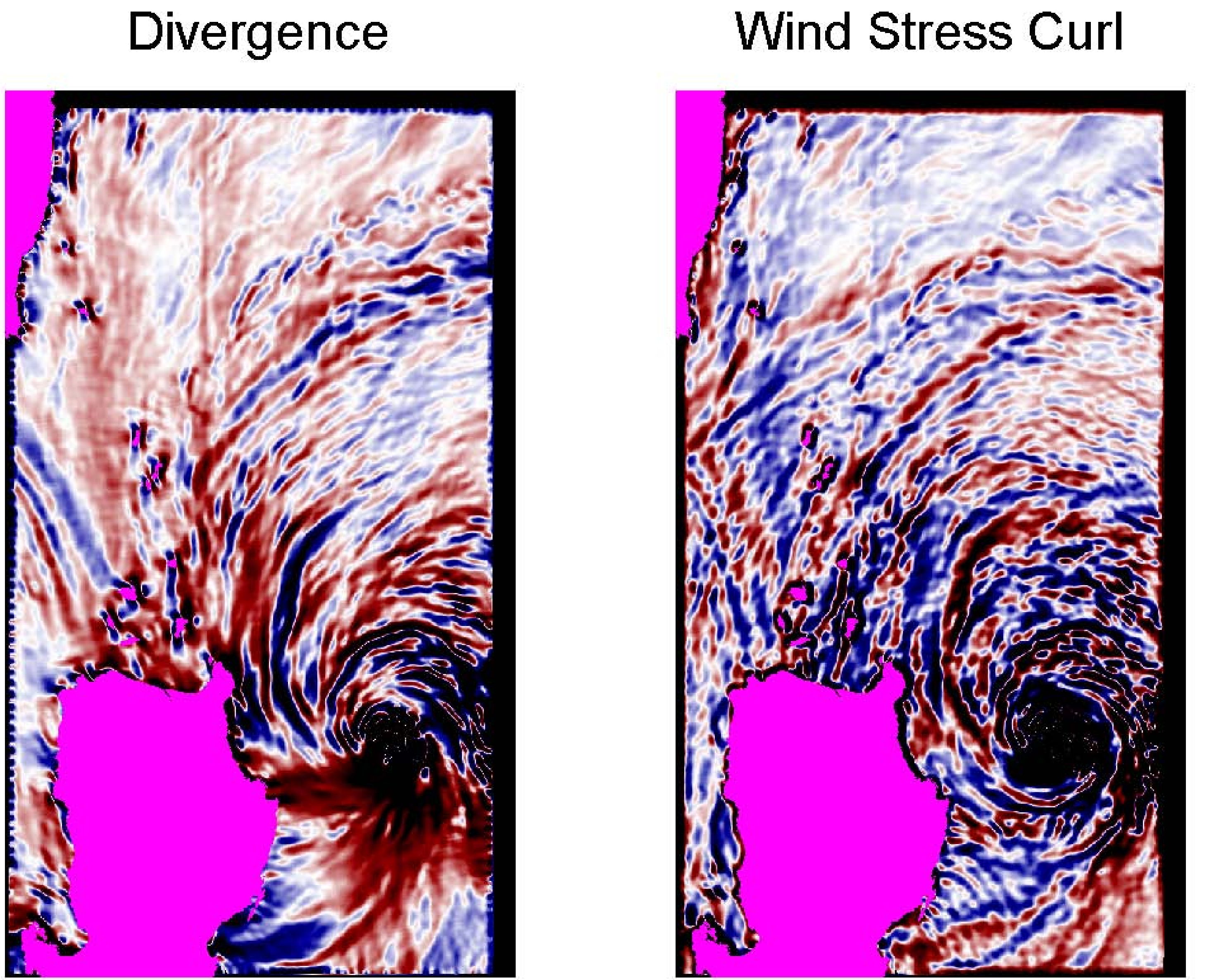
333 m resolution numerical model
Consistent feature in the simulations

133 km box
15 m Divergence field

Motivation for Study

- Can this signal be used to improve SAR (or UHR scatterometer) surface wind retrievals?
- Do they affect PBL fluxes?
- Do they affect air-sea interaction?
  - Wind stress curl?

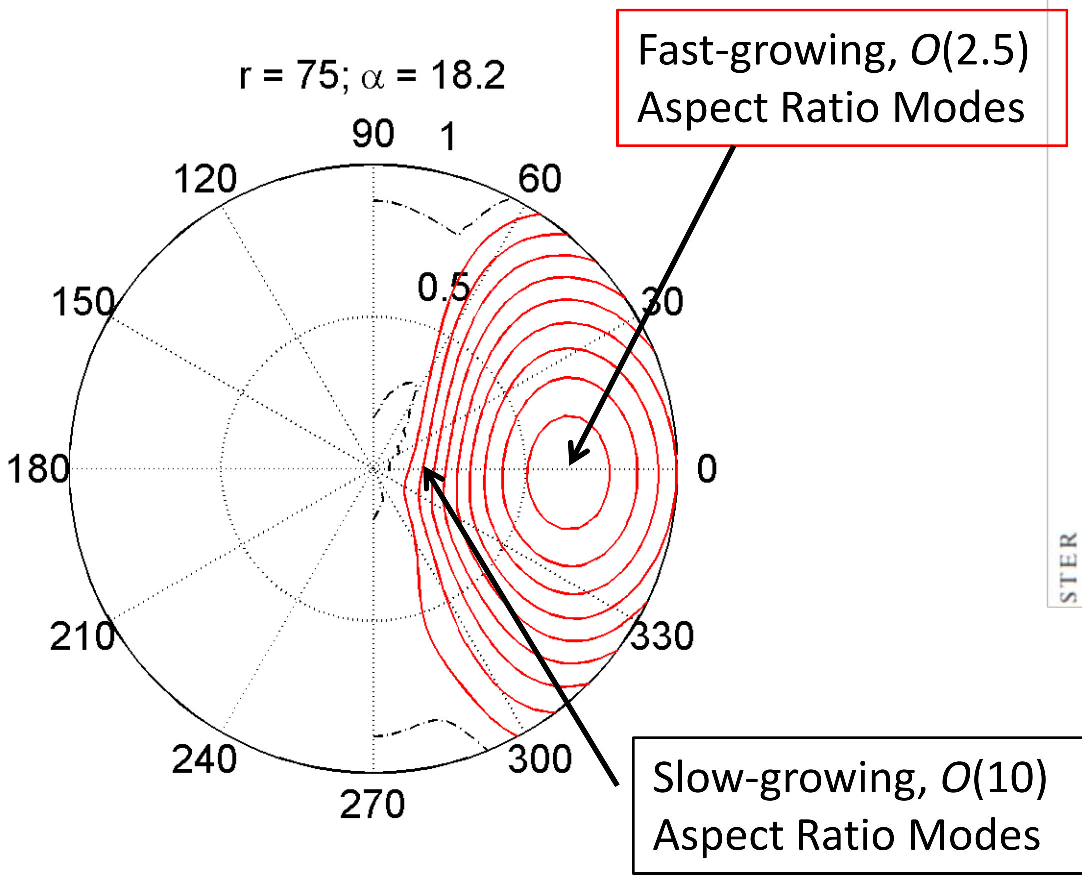
Signature may be clearer in (ρ=const) WSC than in DIV



Summary

- All SAR TC scenes show surface wind organization at *O*(10 km) wavelength; consistent with large aspect ratio PBL rolls
  - 1 km SAR wind pixels (from 25 m σ_0 pixels); SLP-filtered winds
  - Consistent with Gall et al. (1998) radar data and brand-new Gruskin & Tripoli numerical modeling research
- A resonant triad wave-wave interaction model shows that low-order phase coupling feeds energy from the dominant rolls into the slowly-growing long wavelength rolls
  - TC PBLs nearly always form *O*(2 km) wavelength rolls
  - Simple and reasonable possible mechanism
- Future work
  - Extend to non-co-linear waves (string of pearls)
  - Cold-air outbreaks

Extra Slides



Theory shows dominant
modes match with observations
~sub-km to 2 to 3 km

Large aspect ratio modes are too slowly-growing
To compete with lower aspect ratio modes

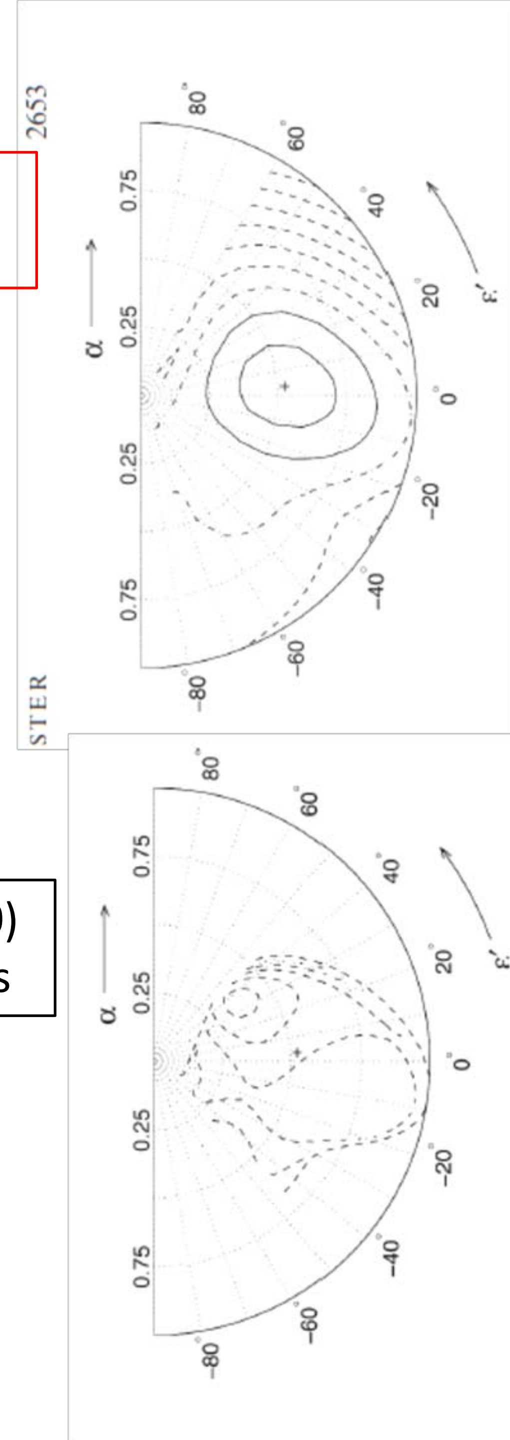
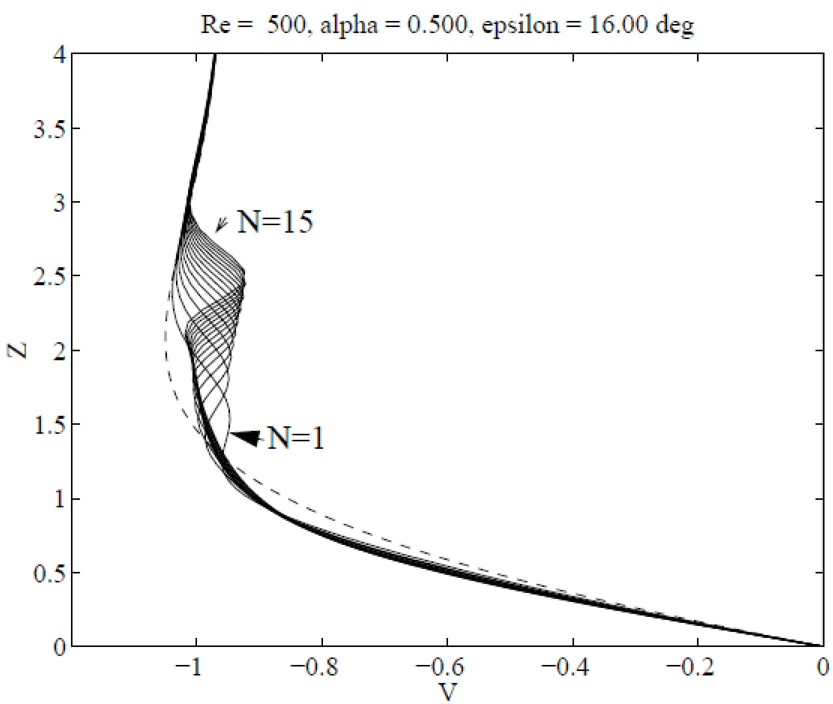
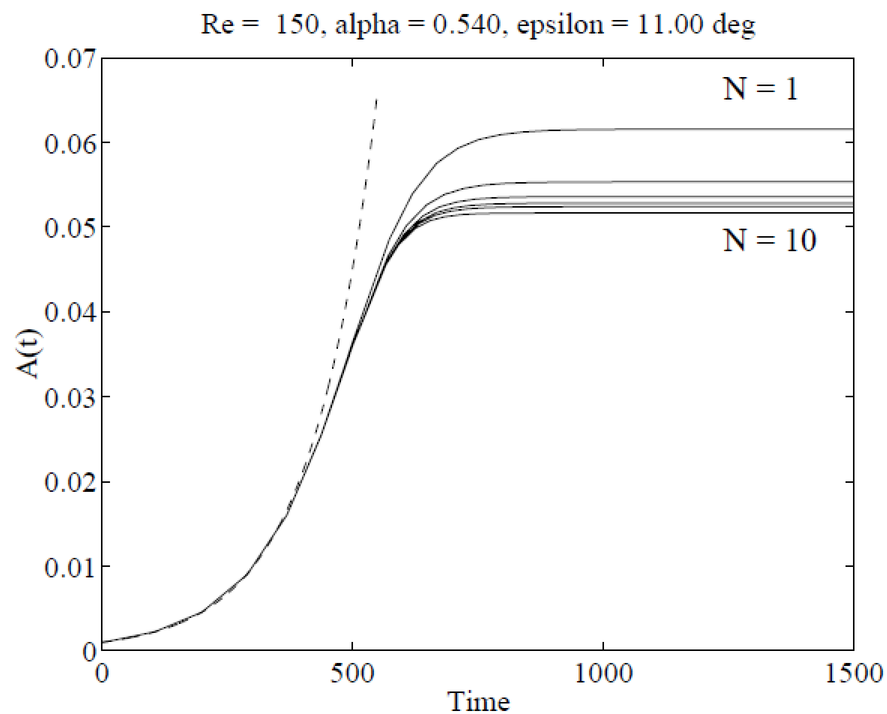


FIG. 7. First Landau coefficient a_1 corresponding to the conditions in Fig. 4 for $a_0 > 0$. The $a_0 = 0$ contour is shown as dash–dot. Contour interval is 0.0005 (nondimensional).

Low-Order Truncation Errors

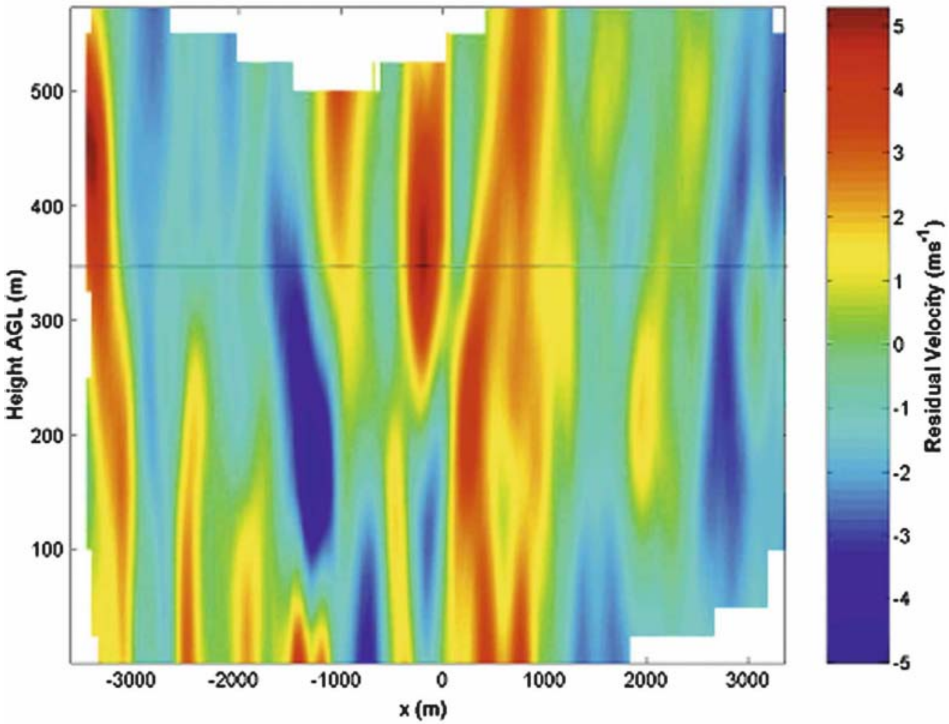


Mean-flow Modification



Amplitude Estimation

Transfer Moderate (~2 km) to Small (sub-km) Scales



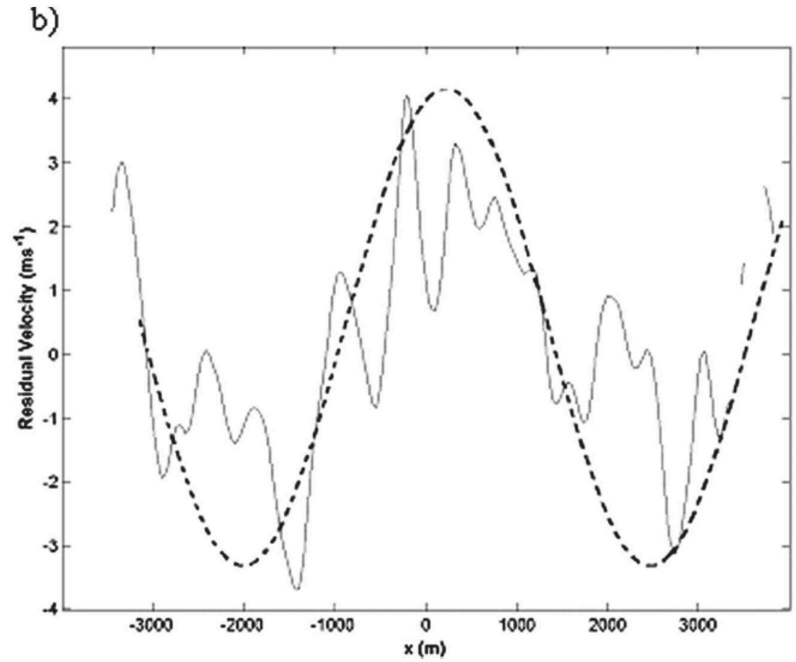


FIG. 16. Example of superposition of scales of motion. (a) Vertical cross section of residual radial velocity from Hurricane Frances. The solid line indicates the height at which the data presented in (b) were extracted. (b) The thin line represents the individual residual velocity data at 350 m AGL, and the dashed line outlines the larger scales of motion superimposed on the signal.

An Observational Study of Hurricane Boundary Layer Small-Scale Coherent Structures

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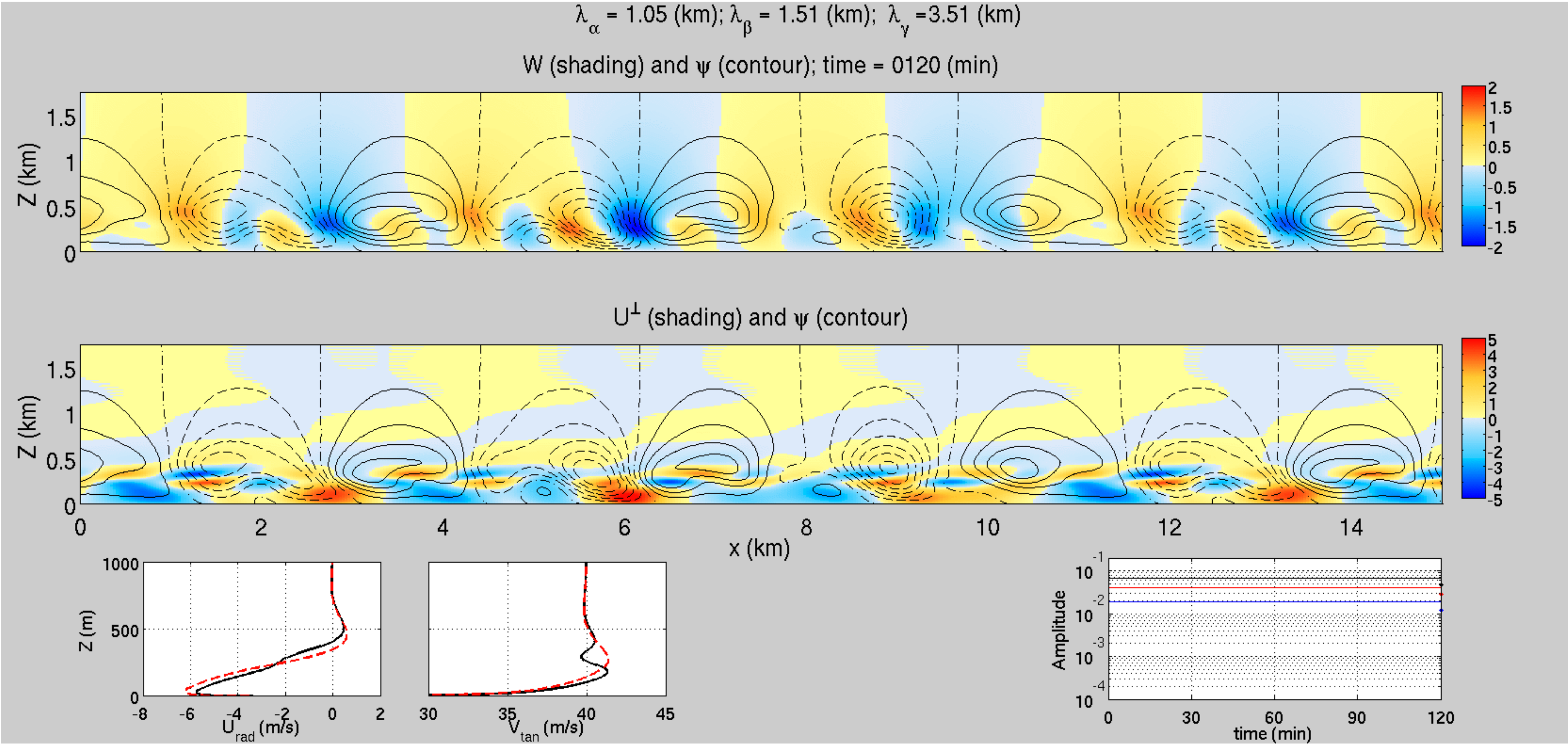
(Manuscript received 4 June 2007, in final form 7 January 2008)

ABSTRACT



λα = 1.05 (km); λβ = 1.51 (km); λγ =3.51 (km)
W (shading) and ψ (contour); time = 0120 (min)
Z (km)
U⊥ (shading) and ψ (contour)
x (km)
Z (m)
Urad (m/s)
Vtan (m/s)
Amplitude
time (min)

dicular to the rolls.

Radar volumes were updated every 300 s; these intervals were too long to permit esti-

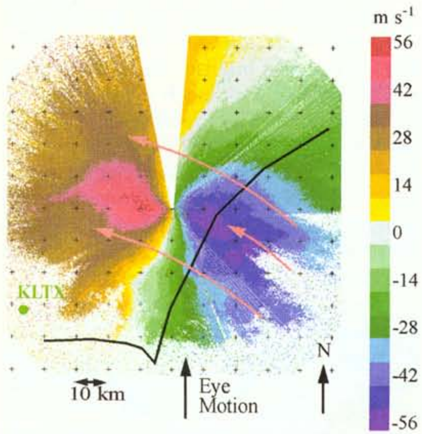


Fig. 3. Large-scale Doppler velocity structure at 23:30:19 UTC, as measured by the DOW radar. Strong easterly flow peaking at ~60 m s^{-1} is evident both off- and onshore. The eye of the hurricane is at the edge of radar visibility to the south. Visibility was severely limited by attenuation. Pink curved arrows illustrate average wind flow. Scan is at 5° elevation.

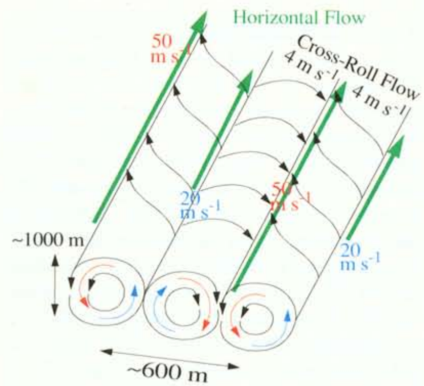


Fig. 5. Schematic representation of observed shear- and wind-parallel boundary layer rolls. High-momentum air (red) is brought to the surface in the downward legs of the rolls, while air slowed near the surface is brought aloft in the upward legs.

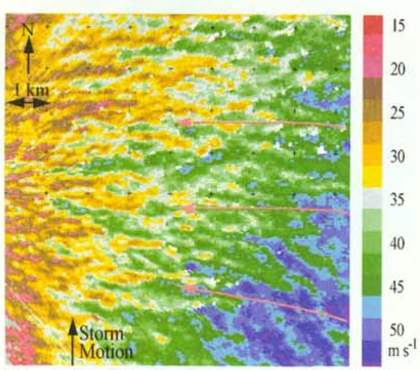


Fig. 4. High-resolution image of Doppler velocity field to the east of Wilmington at 23:58:17 UTC. Sub-kilometer-scale streaks caused by boundary layer rolls modulate the mean easterly flow. Near the radar (left) at altitudes of ~100 m agl, peak and trough wind speed values are ~40 m s^{-1} and ~10 m s^{-1}, respectively. Further from the radar (right), peak and trough wind speed values alternate from ~25 to ~55 m s^{-1}. Azimuthal shear values are (~30 m s^{-1}/~300 m) ≈ 0.1 s^{-1} across many of the rolls. Scan is at 2° elevation.

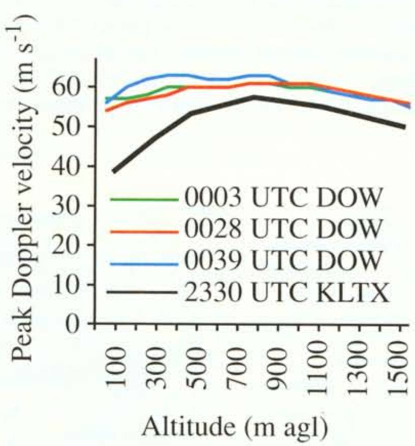


Fig. 6. Altitude dependence of peak wind speeds as observed by DOW and National Weather Service KLTX radars. DOW-measured peak speeds at 100 m agl are nearly as high as those at 1000 m agl as a result of momentum transport in the rolls and agree closely with surface peak wind observations. KLTX-measured peak speeds are smaller at low altitude because of poorer resolution and possibly because of longer overland trajectories.

~30 m/s mean +/- 15 m/s across-roll variation in low-level wind

Wurman and Winslow (1998) *Science*, **280,** 555-557

An Observational Case for the Prevalence of Roll Vortices in the Hurricane Boundary Layer*

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(Manuscript received 5 April 2004, in final form 12 December 2004)

Why Rolls are Prevalent in the Hurricane Boundary Layer

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(Manuscript received 5 April 2004, in final form 1 December 2004)

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