Incorporation of New Nonlinear Similarity PBL Model in Ocean Vector Wind Surface Pressure Retrievals

Ralph Foster, APL, University of Washington

Jerome Patoux, R.A. Brown, Atmospheric Sciences, University of Washington

Goal

- Framework for ***simple*** PBL model that makes best possible use of OVW swath data
  - Nonlinear momentum advection ← (*primary motivation*)
  - Thermal Wind
  - Stratification
  - Non-local momentum flux
  - Entrainment
- Use
  - Improve & unify our various U10N → SLP methodologies
  - Remove ad hoc “gradient wind” correction from our SLP retrievals
  - Frontal dynamics studies

Frontal zones, Storms, Tropical Cyclones

- Momentum advection plays an important role in the PBL dynamics
  - Regions of greatest interest in SLP retrieval
- We know that vertical transport of inflow momentum plays a key role in TC BL jet strength
- Want to generalize Vortex PBL theory to asymmetic TCs for SAR OVW analyses

Basic Idea

- Not an “inverse model”
- Top-down is usual starting point
  - Satellites provide *surface* data
- Bottom-up similarity model
- Define dynamical parameters in terms of OVW data
  - How far can we get?

Some Math

- (Any new email?)

Scales

$$V \sim U_{10}^{N}$$

$$L_H \sim const \cdot 2 \frac{U_{10}^{N}}{\left|\nabla U_{10}^{N}\right|}$$

$$h \sim Vert.$$

$$h_{SL} = \lambda h, \lambda \approx 0.15 << 1$$

$$\mathrm{Re} = \frac{\phi\left(\frac{\lambda h}{L}\right)}{k\lambda\sqrt{C_D^N}}$$

$$R_o = \frac{U_{10}^{N}}{hf}$$

$$a = \frac{\mathrm{Re}}{Ro}, (<\approx 2)$$

$$\varepsilon = \mathrm{Re}\frac{h}{L_H} < 1$$

Simple Similarity

$$\hat{u} \sim U_{10}^{N}(x,y)\, y_1(z)$$

$$u' \sim \frac{U_{10}^{N}(x,y)}{h}\, y_2(z)$$

$$\hat{v} \sim U_{10}^{N}(x,y)\, y_3(z)$$

$$v' \sim \frac{U_{10}^{N}(x,y)}{h}\, y_4(z)$$

$$w \sim \frac{h}{L_h} U_{10}^{N}(x,y)\, y_5(z)$$

$$z = \frac{\hat{z}}{h}$$

$$H_x = \frac{h}{L_h}\frac{\partial h}{\partial x}$$

$$\frac{\partial u}{\partial x} = U_x y_1 - z H_x y_2$$

$$U_x = \frac{U_{10}^{N}}{L_h}\frac{\partial \left|U_{10}^{N}\right|}{\partial x}$$

Two Lower Boundary Conditions from OVW (non-dimensional)

$$U'(0) = C_D^N \operatorname{Re} \qquad \overset{\square}{U}(0) = b_3$$

$$b_3 = \left[1 + \frac{\sqrt{C_D^N}}{k} \left(\log \frac{\lambda h}{10\,m} - \psi\left(\frac{\lambda h}{L} \right) \right) \right]$$

$$b = \frac{C_D^N \operatorname{Re}}{h b_3}$$

(Simple restatement of Monin-Obukhov: U’ = bU)

Simplest Case for Testing (Ekman)

- Midlatitudes
- Constant K
- Neutral stratification
- Barotropic
- No nonlocal fluxes
- No entrainment
- $H_x = \left(1 + \frac{U_{10}^N}{C_D^N} \frac{dC_D^N}{dU_{10}^N} \right) U_x$
- Framework <u>easily</u> generalizes to include all of these effects

Nonlinear terms are $O(\varepsilon)$ …

$$\varepsilon\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right]-a\left[v-V_g\right]=\frac{\partial^2 u}{\partial z^2}$$

$$\varepsilon\left[u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right]+a\left[u-U_g\right]=\frac{\partial^2 v}{\partial z^2}$$

$$-\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=\frac{\partial w}{\partial z}$$

… so, expand solution in ε and decouple nonlinear PDEs into a sequence of linear ODEs:

$$u=\sum_{n=0}^{N}\varepsilon^n y_{1_n}$$

"Nonlinear" forcing at O(n) is determined by O(n-1) solutions:

$$u\frac{\partial u}{\partial x}=\sum_{k=0}^{n-1}\left[y_{1_k}\left(U_x y_{1_{n-1-k}}-zH_x y_{2_{n-1-k}}\right)\right]$$

Forced Ekman

$(q = y_1 + iy_3)$

$$q_n'' - iaq_n = -iaq_{g_n} + F_n$$

n^{th} contribution

Depends on forcing from max(O(n-1)) terms

$$q = q_h + q_g + q_p$$

homogeneous

Forced by F(z)

Assume high-order corrections are "orthogonal" to linear solution:

$$q_{n=0}(0) = b_3$$

$$q_{n>0}(0) = 0$$

OVW →Pressure Gradients

Two lower BCs & assumption that $q\left(z_t\right) = q_g$

Gives algebraic solution for geostrophic wind, given OVW:

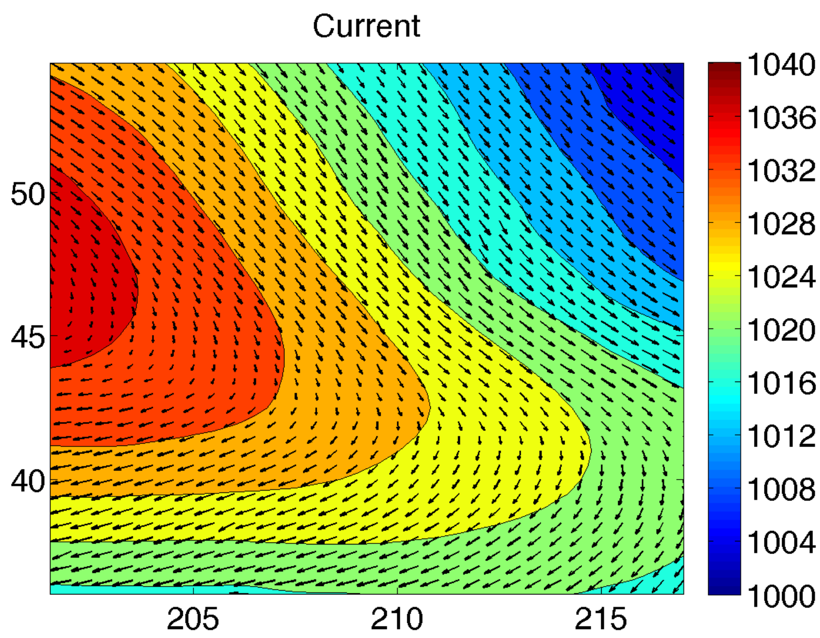
$$q_{g_0} = \left[1 + \frac{b}{B}\tanh\left(Bz_t\right)\right] b_3 U_{10}^N \Rightarrow b_3\left(1 + \frac{b}{B}\right) U_{10}^N$$

$$q_{g_n} = q_{p0} - \frac{q'_{p0}}{B}\tanh\left(Bz_t\right) + \frac{q_p\left(z_t\right)}{\cosh\left(Bz_t\right)}$$

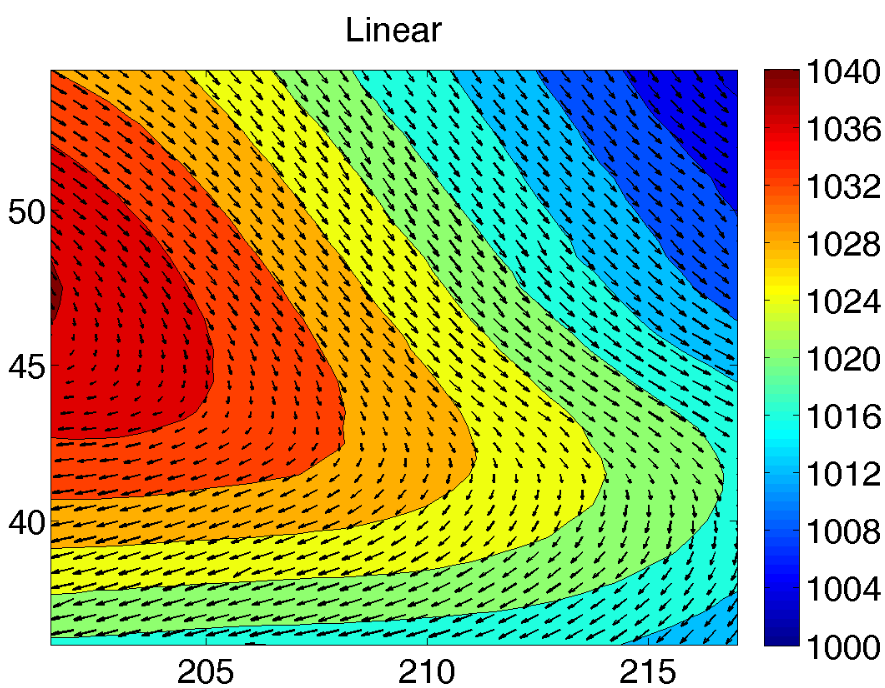
$$\left(B = \sqrt{ai} \,\square\, 1 + i\right)$$

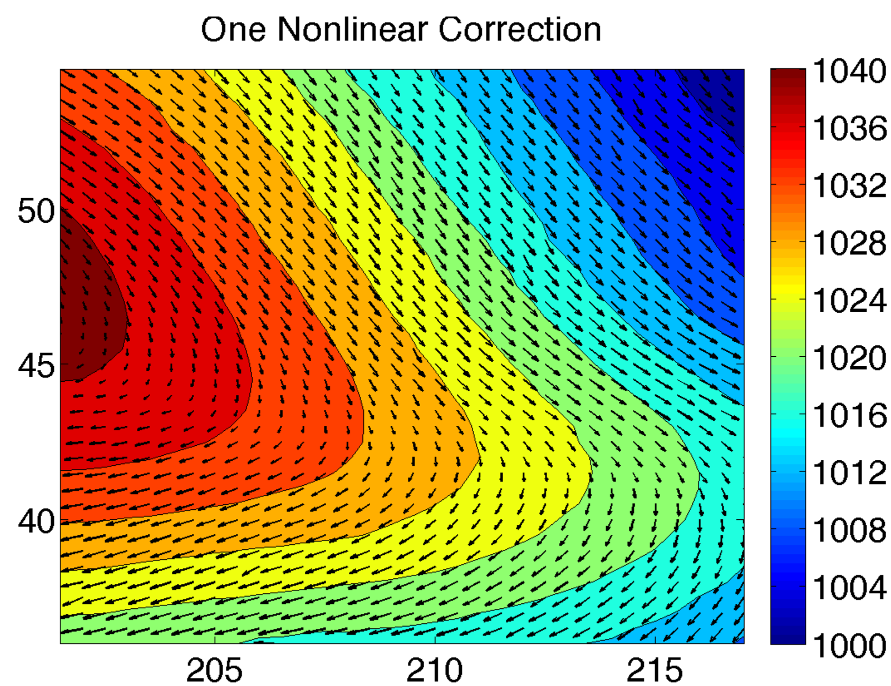
Note

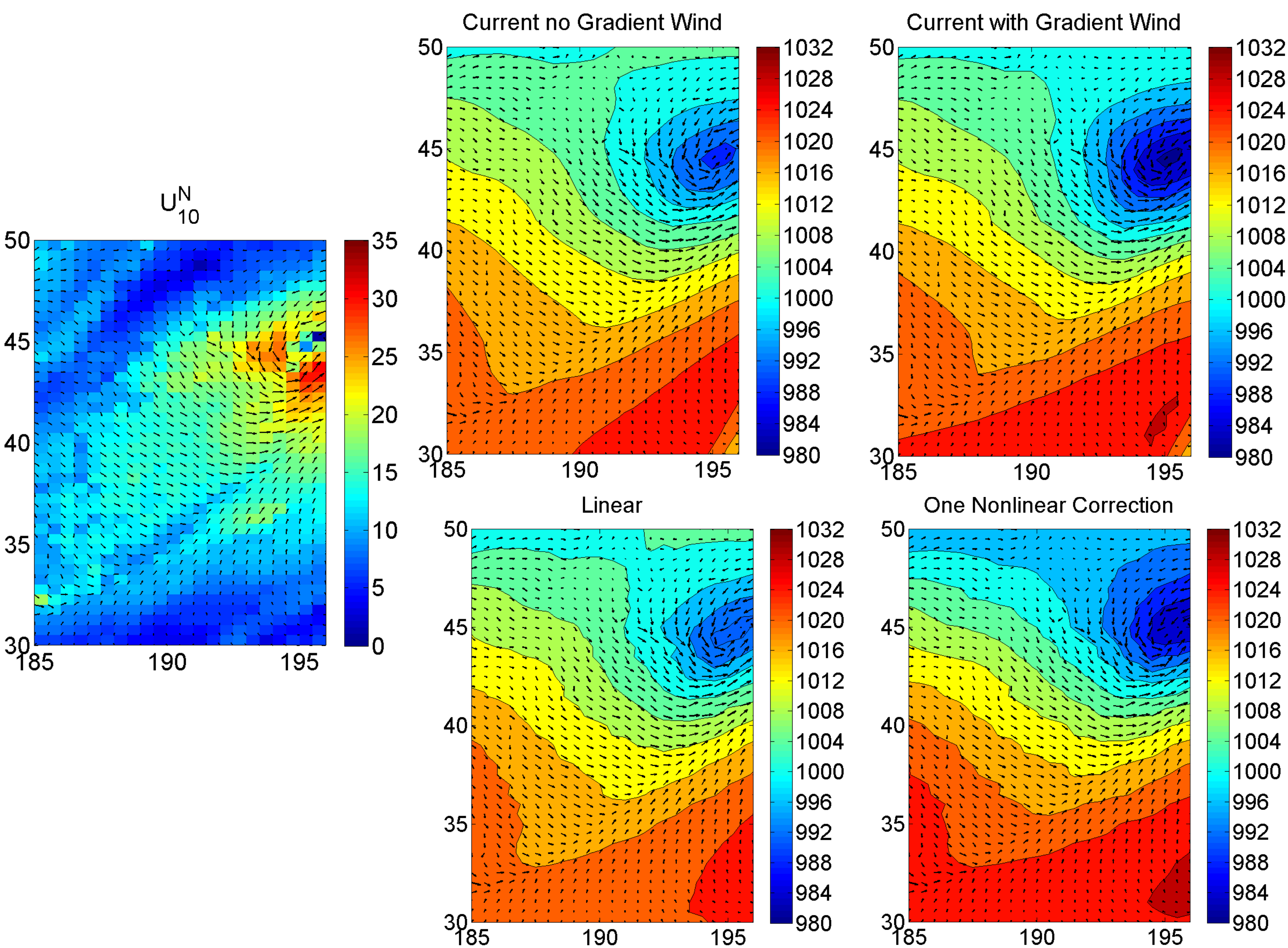
- Linear (0^{th}-order) shows direct relationship between OVW and the dominant contribution to the pressure gradients given reasonable assumptions and perhaps ancillary data for stratification and/or thermal wind effects
  - 1 + b/B: b is M-O similarity, B = 1 + i for Ekman
  - This is why SLP retrievals have proven so accurate
  - No need for “inverse” model to get much of grad(P)
- “Higher-order” corrections to pressure gradients only require basic ODE solvers (as long as one doesn’t go to very high order).



Some important differences remain.
New model should include stratification for a fairer comparison.
Predict minimum of 2 NL corrections are needed.







U_{10}^{N}
Current no Gradient Wind
Current with Gradient Wind
Linear
One Nonlinear Correction

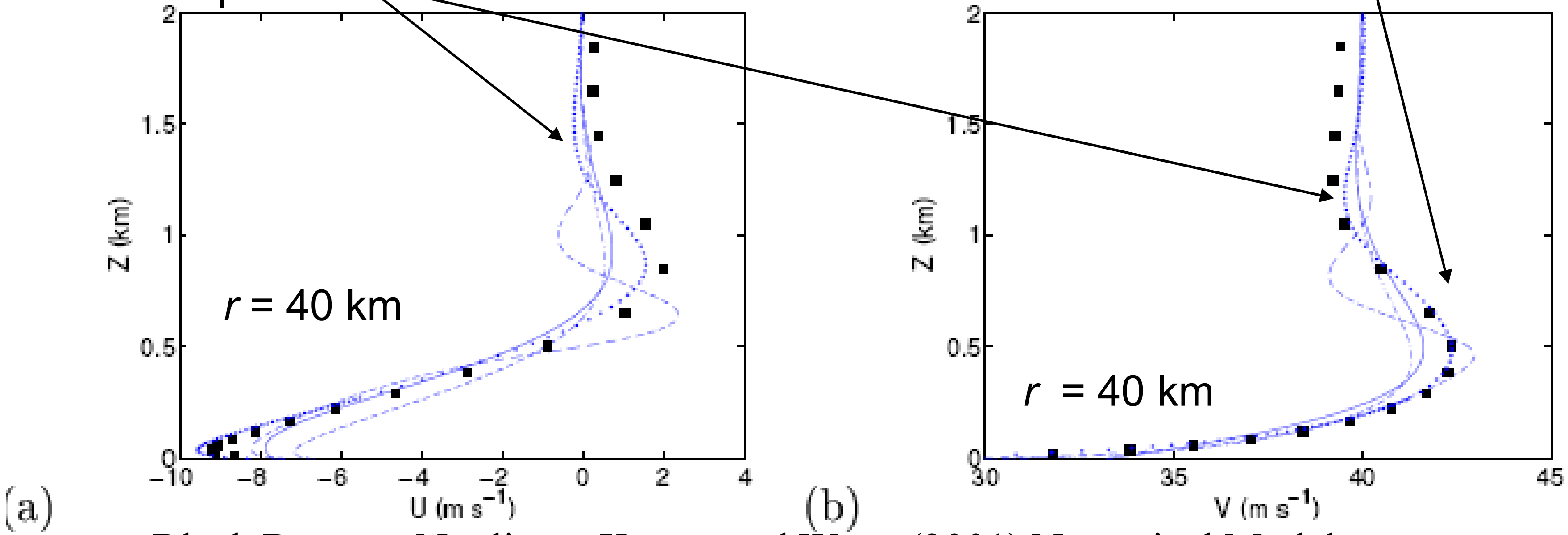
Comments

- Expect at least Two nonlinear corrections are needed
  - E.g. Quasi-Geostropic (one NL term) scaling misses asymmetry between cyclones and anti-cyclones
- For Ekman case, simple formulas
  - Useful for analyzing solution characteristics
  - Can still easily include stratification and thermal wind
  - Adding stratification & thermal wind should match current model
  - Variable K of interest
  - Non-local fluxes of interest

"*Backwards*" PBL Model: grad(P) held constant, wind allowed to vary

$$w\frac{\partial U_{rad}}{\partial z}$$

Different *K(z) leads to* different profiles

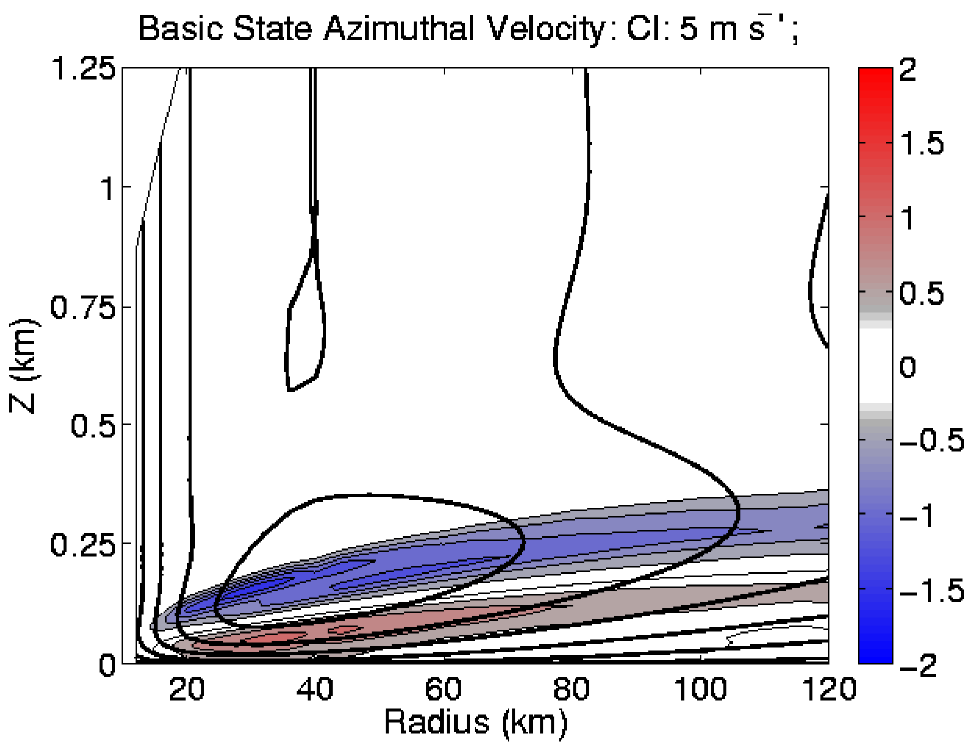


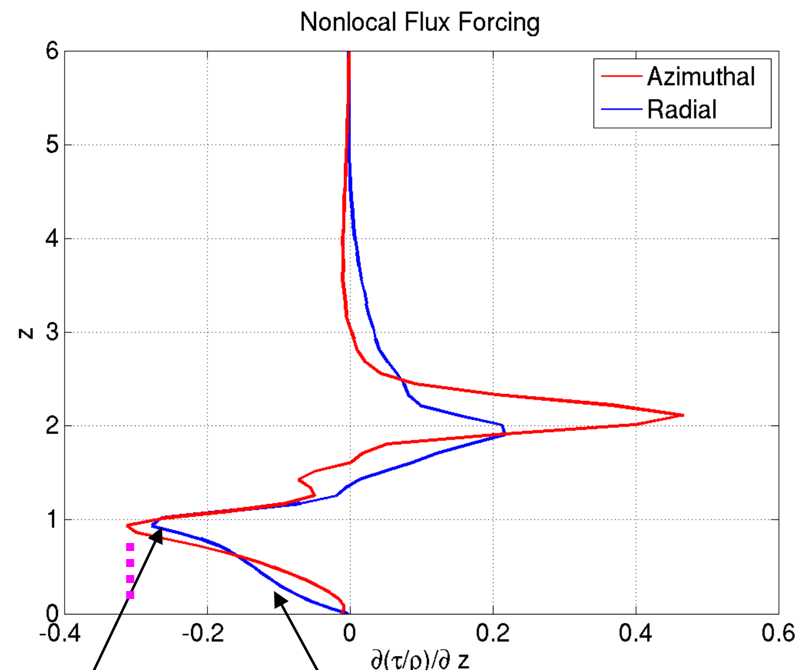
Black Dots are Nonlinear Kepert and Wang (2001) Numerical Model

Blue dotted line is Similarity Model Driven by Kepert & Wang K(z) (39 m/s)

→ Similarity Model reproduces results of time-stepping numerical model!

Nonlocal Fluxes





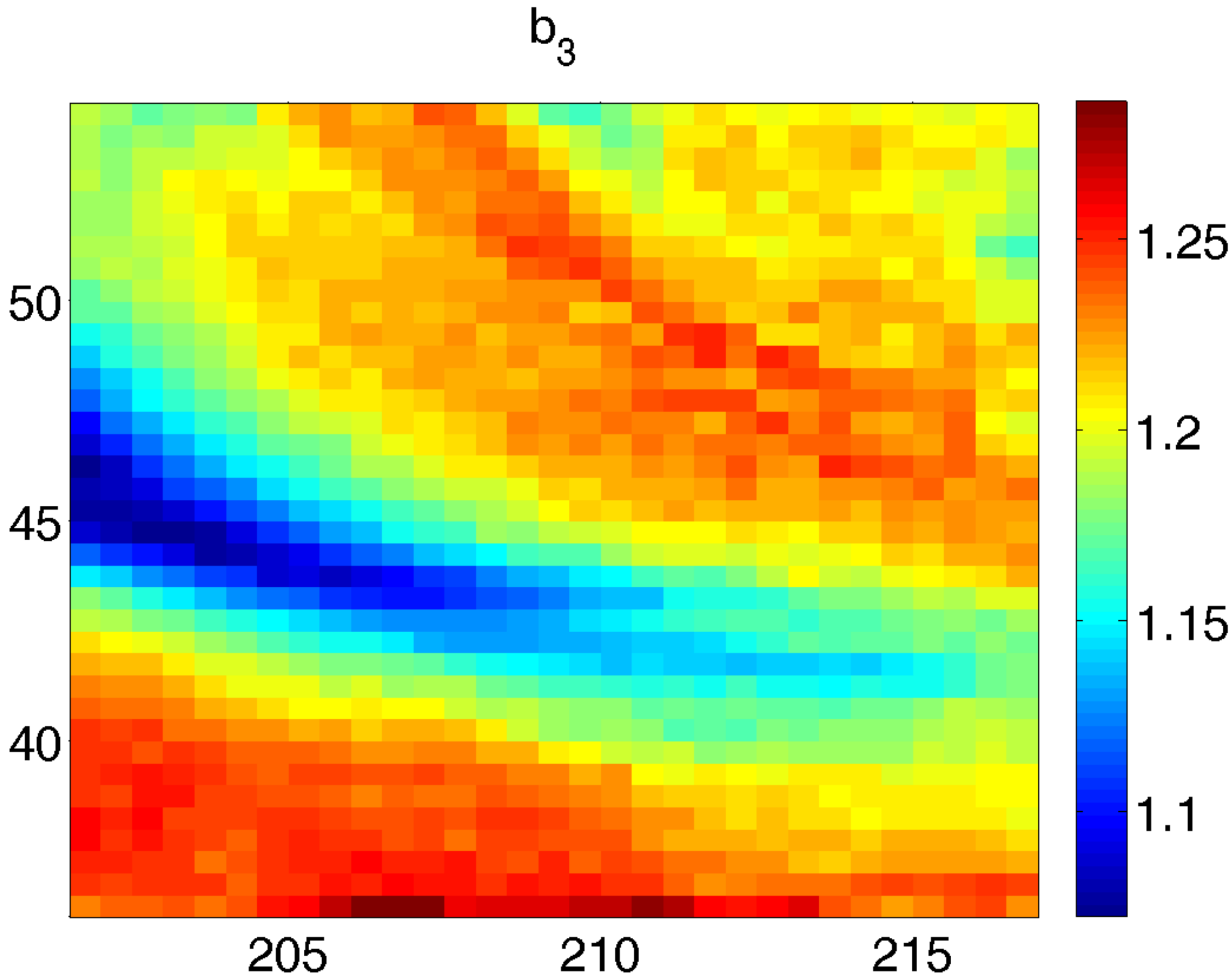
New model fixes
Near-surface

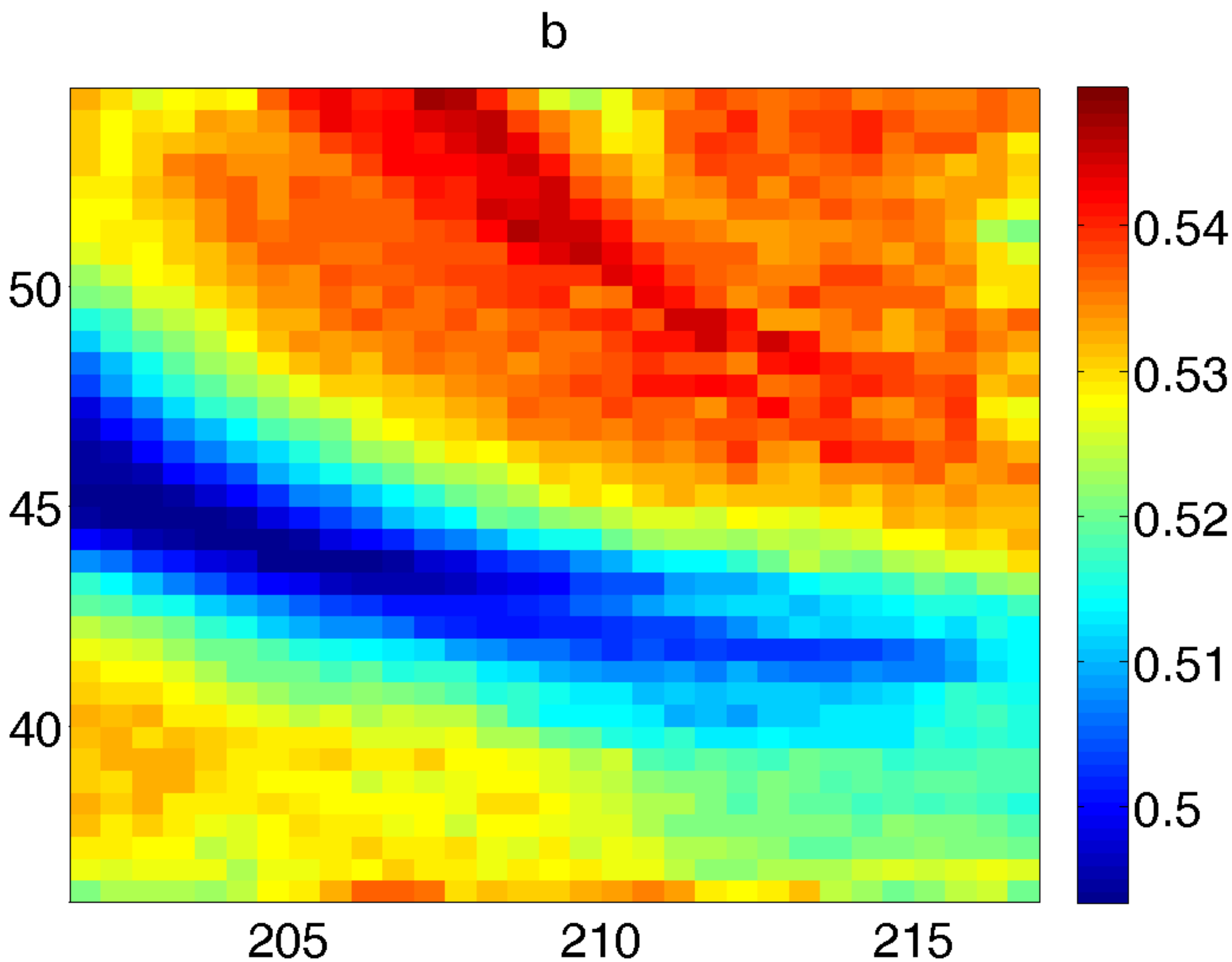
$$\frac{\partial \tau}{\partial z} = \frac{\partial \tau_{local,existing\ PBL\ param.}}{\partial z} + \underbrace{\frac{\partial \tau_{non-local}}{\partial z}}$$

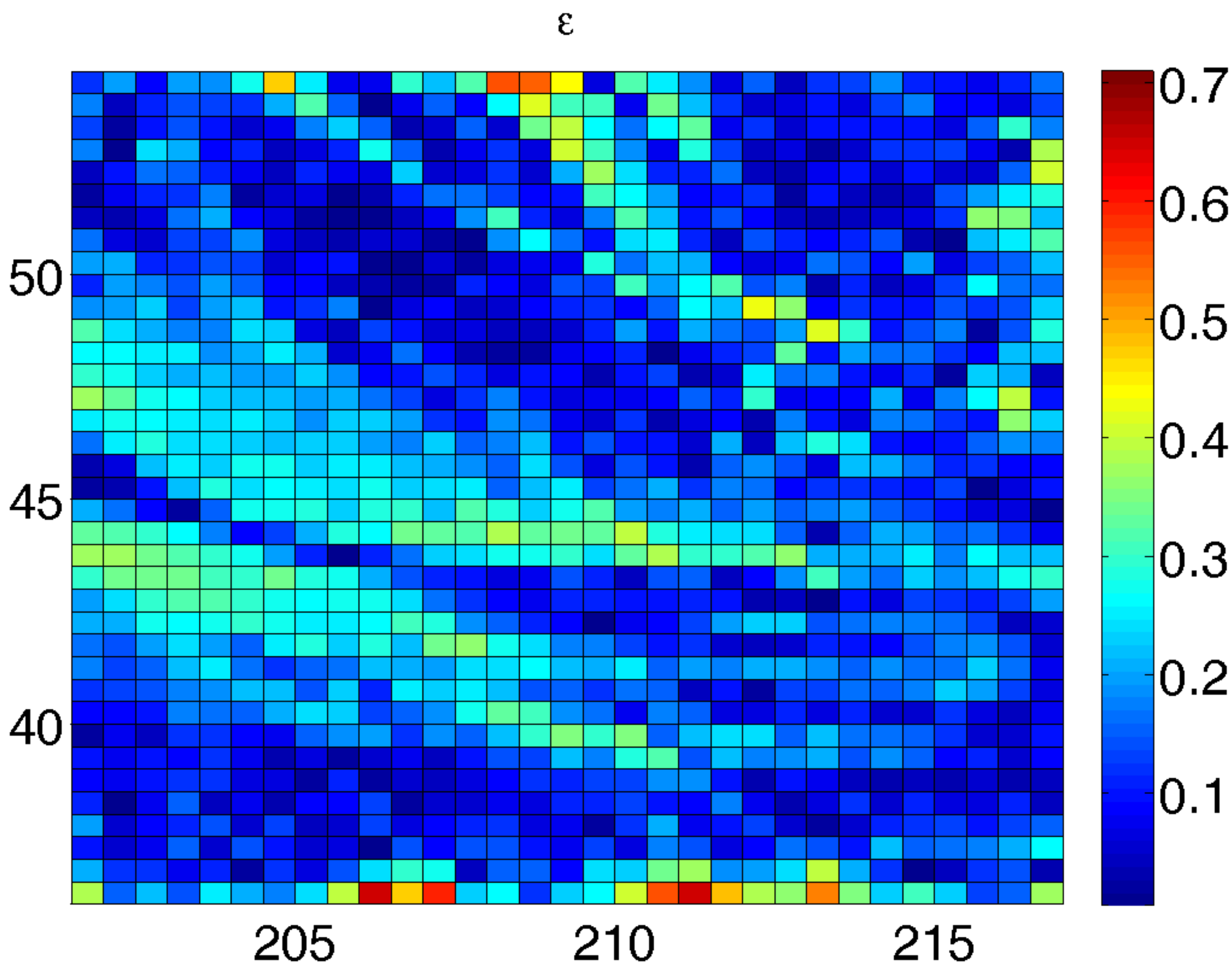
Upcoming Year

- Help Jerome implement new model in "production" SLP code.
- Repeat Patoux et al. (2008)-like study using buoy bulk pressure gradients & ECMWF analyses
  - Evaluate new model against old
  - Tune a few parameters & examine convergence
  - Examine fun and informative cases (e.g. fronts & tropical cyclones)

Parameters


b_3
50
45
40
205
210
215
1.25
1.2
1.15
1.1


b
50
45
40
205
210
215
0.54
0.53
0.52
0.51
0.5


ε
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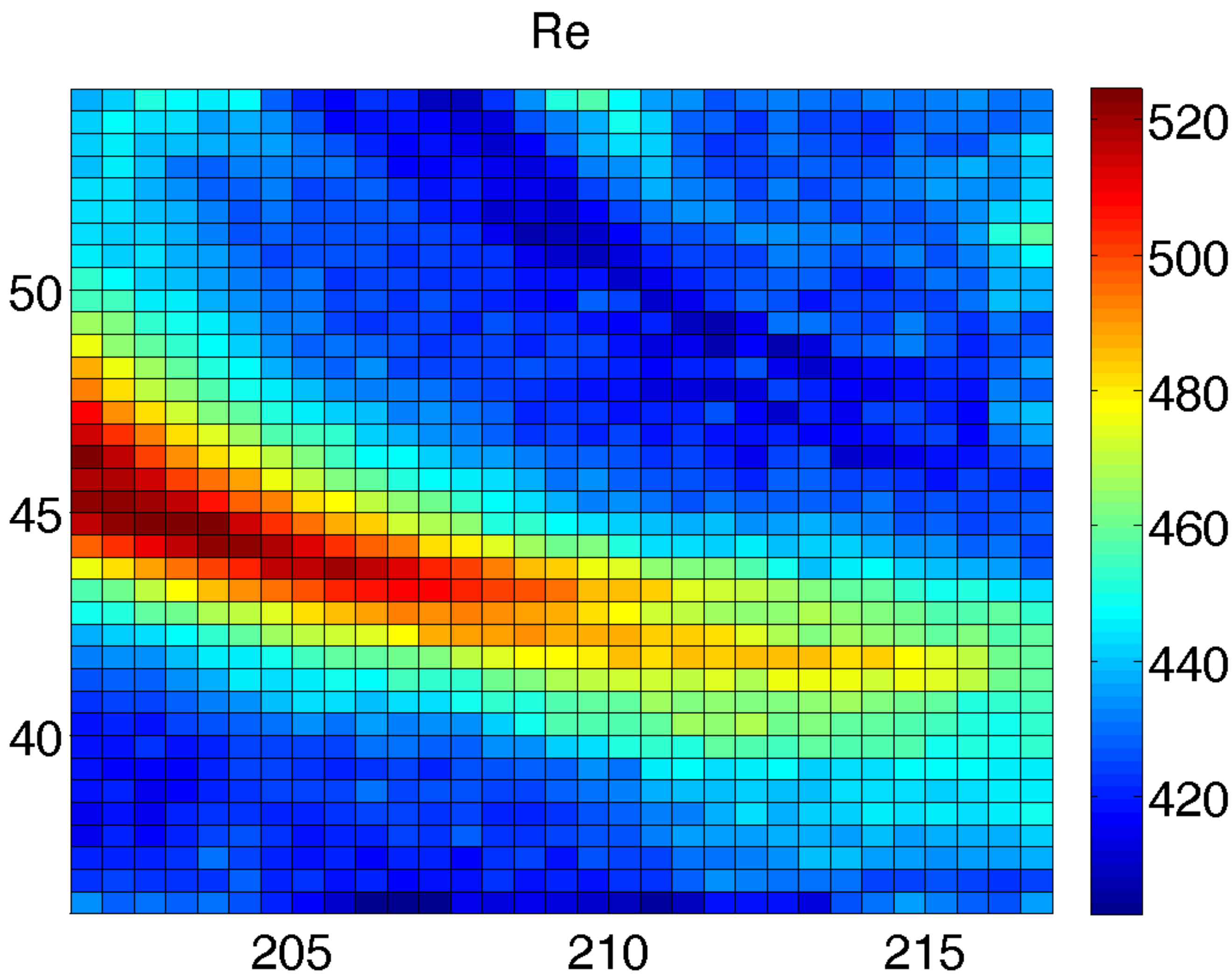
45
40
205
210
215
0.7
0.6
0.5
0.4
0.3
0.2
0.1


Re
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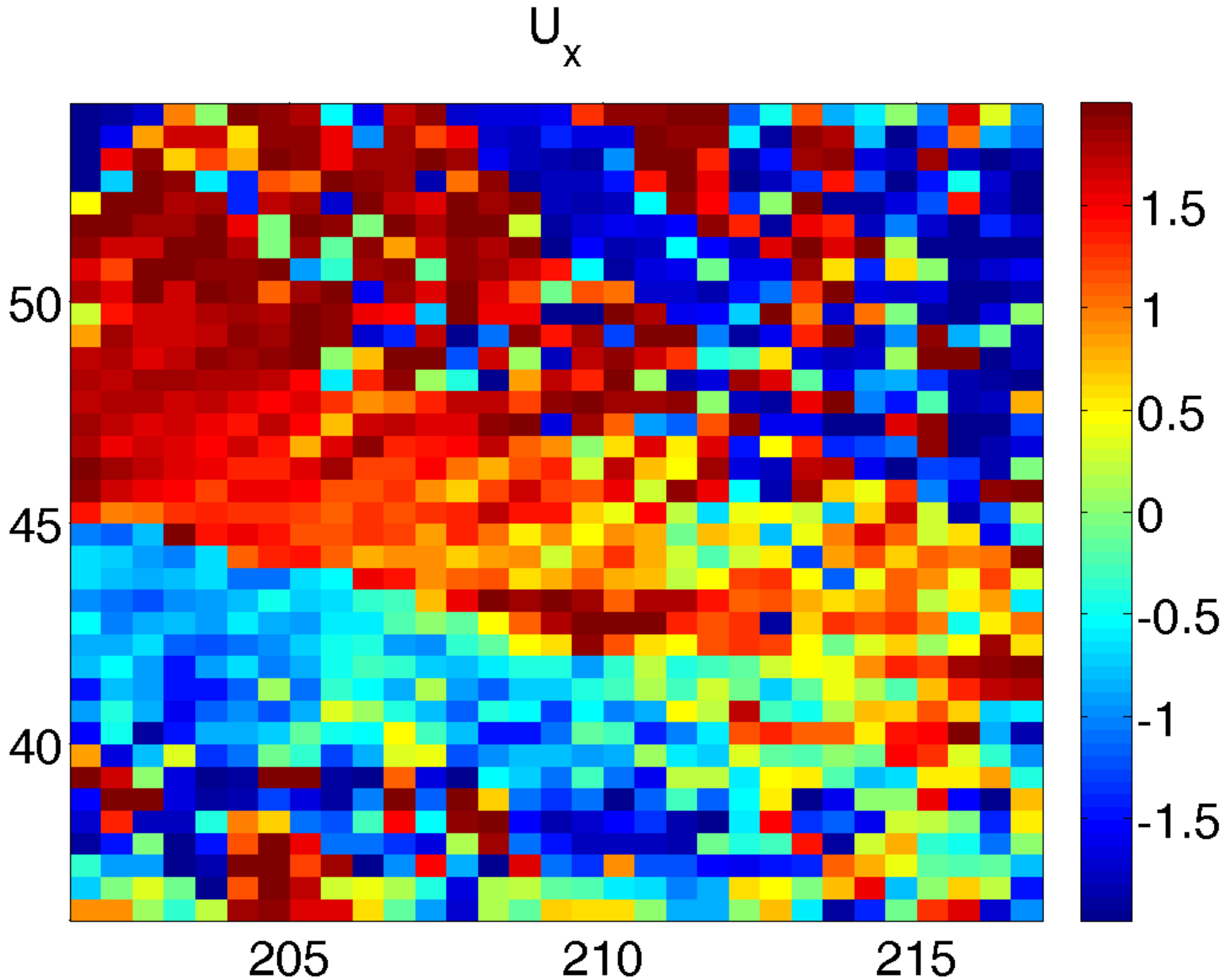
45
40
205
210
215
520
500
480
460
440
420



U_x
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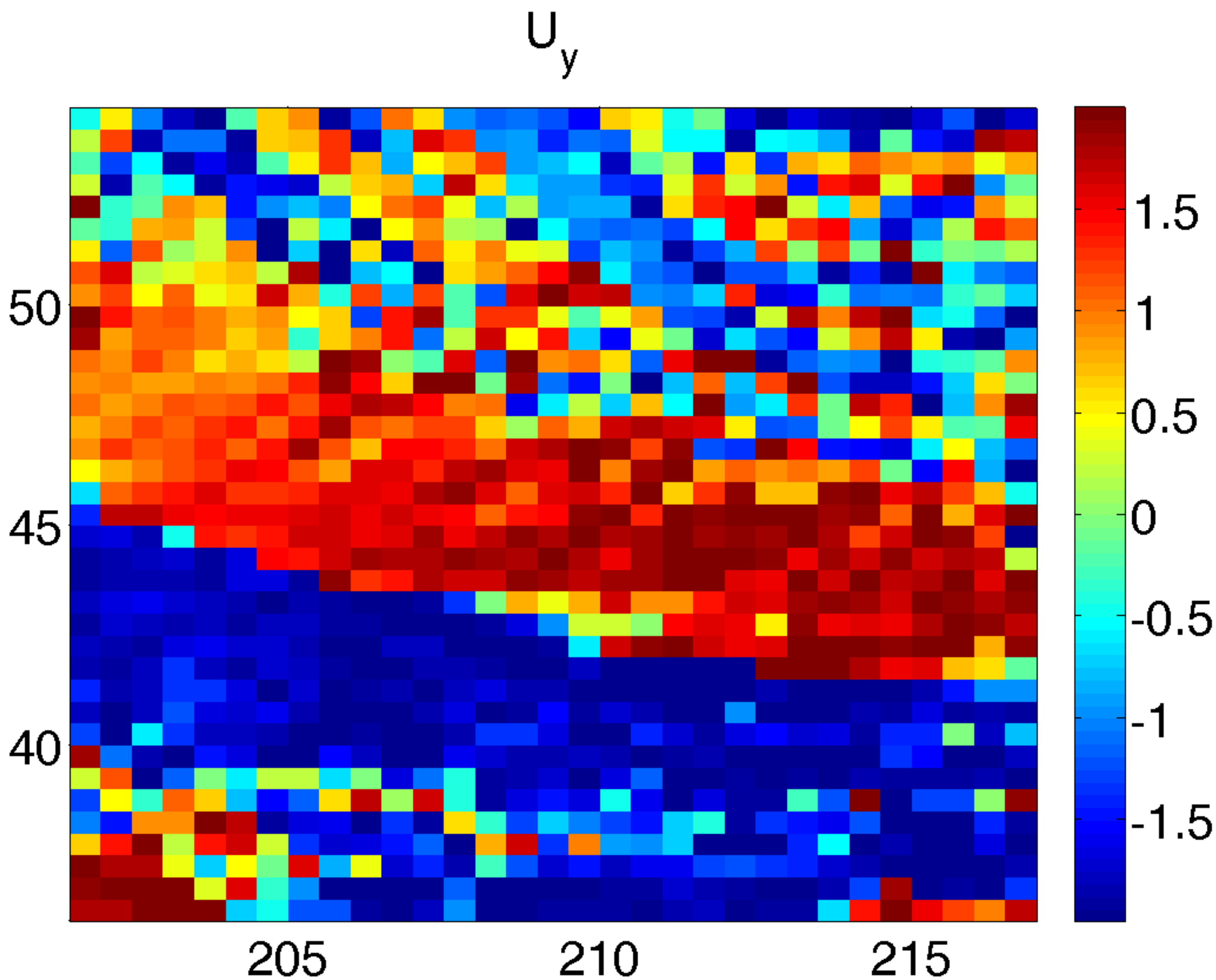
45
40
205
210
215
1.5
1
0.5
0
-0.5
-1
-1.5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210
215
1.5
1
0.5
0
-0.5
-1
-1.5