Quantifying QuikSCAT Value-Added in Estimates of the Surface Wind Field over the Mediterranean Sea on Ocean Forecast Timescales

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- *Review BHM*
- *MFS application*
- *Uncertainty in Posterior Distributions and Spectra*
- *Uncertainty as f(x,y,t)*
- *Goals/Plans: Physical Interpretation in BHM*

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MFS-Wind-BHM;

ensemble surface vector winds (SVW), for ensemble data assimilation and forecasts

Strategy:

Exploit abundant and precise observations, and precisely characterized uncertainty in QuikSCAT SVW retrievals, to deduce and distribute ocean forecast model uncertainties (less well known).

SVWs are an effective means of identifying uncertainties in many dependent variable fields of the ocean forecast model.

Credit: Alessandro Bonazzi

Background:

Labrador Sea Implementation with NSCAT (Royle et al., 1998)

Tropical Wind Implementations (Wikle et al, 2001; Hoar et al, 2003)

Multi-platform OSSE (Berliner et al., 2003)

Mediterranean Applications (Milliff et al., 2008; Bonazzi et al., 2008; Wikle et al., 2009; ....)

Review: Bayesian Hierarchical Models (BHM)

Probability Models:

Conditional thinking; [A,B,C] = [A | B,C] [B | C] [C], easier to specify conditional vs joint
Use what we know/willing to assume to simplify; e.g. [A | B,C] = [A|B]

BHM Building Blocks:

Data Stage Distribution (likelihood) quantifies uncertainty in relevant observations, e.g. cal/val information for QuikSCAT $[S \mid U, V, \ldots, \theta_d]$

Process Model Stage Distribution (prior) quantifies uncertainty in (perhaps incomplete) physics of process; e.g. For SVW, $[U_{t+1} \mid U_t, P, \ldots, \theta_p]$

Parameter Distributions from Data Stage and Process Models (i.e. $[\theta_d]$, $[\theta_p]$ *) issues of identifiability, uncertainty, model validation*

Bayes Theorem *relates Data and Process Model Stages to the Posterior Distribution*

BHM Posterior Distribution:

Obtained via Gibbs Sampler Algorithm, Markov Chain Monte Carlo
Best estimates of process (and parameters) given data e.g. $[U, V, P, \ldots, \theta_d, \theta_p \mid S \ldots]$
Posterior mean is best single estimate
Standard deviation of posterior is an estimate of the spread

Prior for the SVW Process:

$$\frac{\partial u}{\partial t} - f\,v = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} - \gamma\,u$$
$$\frac{\partial v}{\partial t} + f\,u = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} - \gamma\,v$$

Choice: begin with RFE
linear, includes surface friction
dependent variables match well with "obs"
include a "friction" process w/o spec form

$$\frac{1}{f}\left[\frac{\partial^2}{\partial t^2} + \left(f^2+\gamma^2\right)\right]u + 2\frac{\gamma}{f}\frac{\partial u}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} - \frac{1}{\rho_0 f}\frac{\partial^2 p}{\partial x\partial t} - \frac{\gamma}{\rho_0 f}\frac{\partial p}{\partial x}$$

Expressions for u,v can lead to proper "full conditionals".

$$\frac{1}{f}\left[\frac{\partial^2}{\partial t^2} + \left(f^2+\gamma^2\right)\right]v + 2\frac{\gamma}{f}\frac{\partial v}{\partial t} = \frac{1}{\rho_0}\frac{\partial p}{\partial x} - \frac{1}{\rho_0 f}\frac{\partial^2 p}{\partial y\partial t} - \frac{\gamma}{\rho_0 f}\frac{\partial p}{\partial y}$$

(classic) Scaling relates RFE to BVE, so "leading order terms" can be identified

A1:

$$u = -\frac{f}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial y} - \frac{\gamma}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial x}$$

$$v = \frac{f}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial x} - \frac{\gamma}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial y}$$

Choice: approximate models ("leading order")
reduce no. terms,
concentrate data stage inputs to reduce uncertainty

A2:

$$u = -\frac{f}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial y} - \frac{\gamma}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial x} - 2\frac{\gamma}{\left(f^2+\gamma^2\right)}\frac{\partial u}{\partial t} - \frac{1}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial^2 p}{\partial x\partial t}$$

$$v = \frac{f}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial x} - \frac{\gamma}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial y} - 2\frac{\gamma}{\left(f^2+\gamma^2\right)}\frac{\partial v}{\partial t} - \frac{1}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial^2 p}{\partial y\partial t}$$

SVW Process Model A1: from Deterministic to Stochastic

1. Discretize RFE approximation (geostrophic-ageostrophic model)

$$u = -\frac{f}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial y} - \frac{\gamma}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial x}$$

$$v = \frac{f}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial x} - \frac{\gamma}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial y}$$

$$U_t = -\frac{f}{\rho_0\left(f^2+\gamma^2\right)}D_y P_t - \frac{\gamma}{\rho_0\left(f^2+\gamma^2\right)}D_x P_t$$

$$V_t = \frac{f}{\rho_0\left(f^2+\gamma^2\right)}D_x P_t - \frac{\gamma}{\rho_0\left(f^2+\gamma^2\right)}D_y P_t$$

2. Add uncertainty term, add levels to model hierarchy (i.e. parameters)

$$U_t = a_1 D_y P_t + a_2 D_x P_t + \varepsilon_u$$

$$V_t = b_1 D_x P_t + b_2 D_y P_t + \varepsilon_v$$

where $\varepsilon_{u,v}$ are <u>error models</u> to be prescribed (we use nested Wavelets)

$$a_1 \approx N\left(-\frac{f}{\rho_0\left(f^2+\gamma^2\right)}, \sigma_{a1}^2\right)$$

$$a_2 \approx N\left(-\frac{\gamma}{\rho_0\left(f^2+\gamma^2\right)}, \sigma_{a2}^2\right)$$

$$b_1 \approx N\left(\frac{f}{\rho_0\left(f^2+\gamma^2\right)}, \sigma_{b1}^2\right)$$

$$b_2 \approx N\left(-\frac{\gamma}{\rho_0\left(f^2+\gamma^2\right)}, \sigma_{b2}^2\right)$$

and the <u>random</u> parameter distributions for a_i b_i are available in the <u>posterior distribution</u>

nb: parameter distributions at another level of the BHM hierarchy

3. Full Conditional Distribution U: Gibbs Sampler

$$[U|\cdot] \;\propto\; [S_t^u|U,\theta_i][A_t^u|U,\theta_i]$$

Data Stage Distributions (QuikSCAT, ECMWF)

$$[U_t|P_t, D_x, D_y, a_i, ...]$$

Process Model Distribution

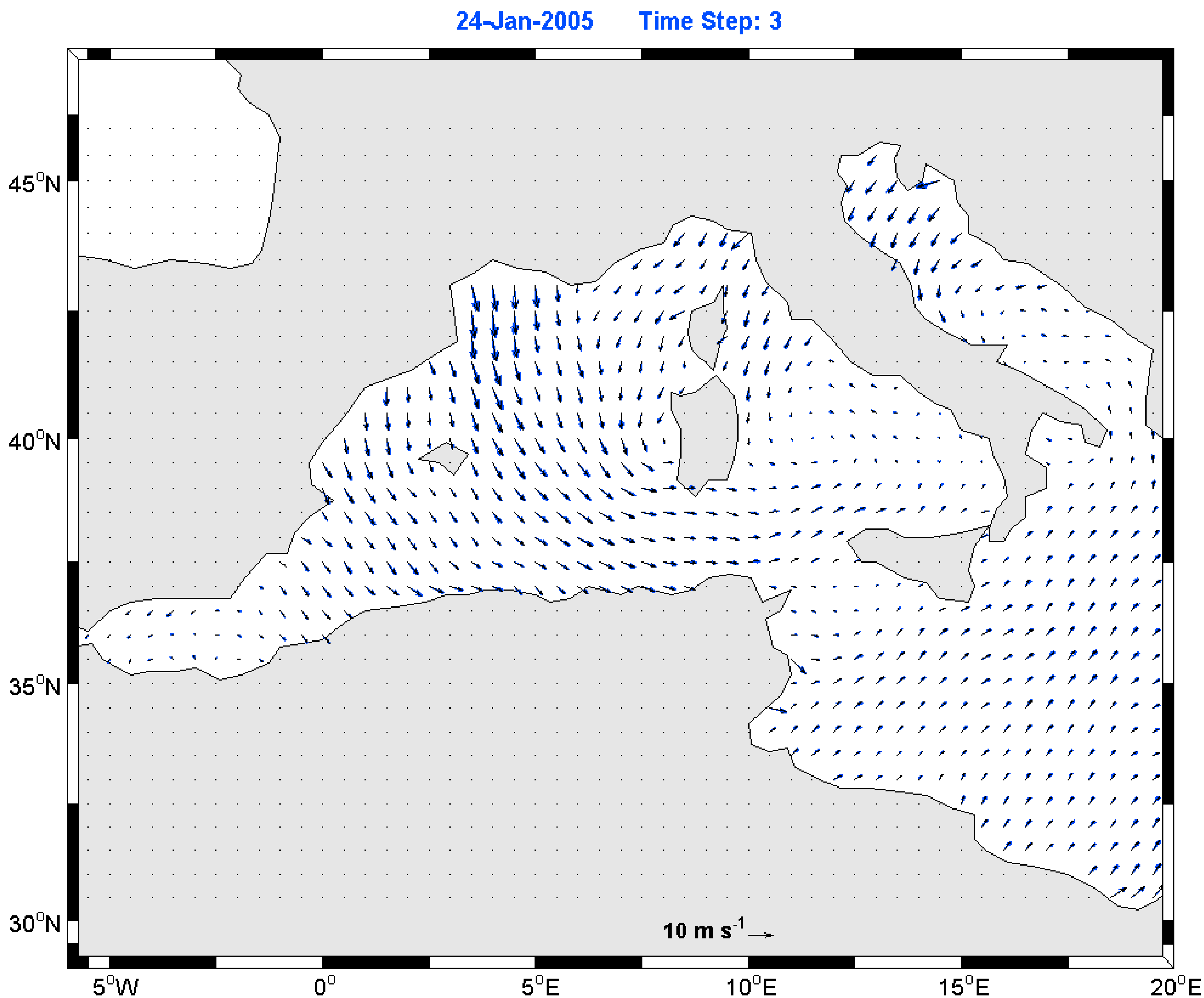
$$[U_t|\cdot] \sim N(A^{-1}B, A)$$

Gaussian Assumption:
less restrictive for conditional distributions

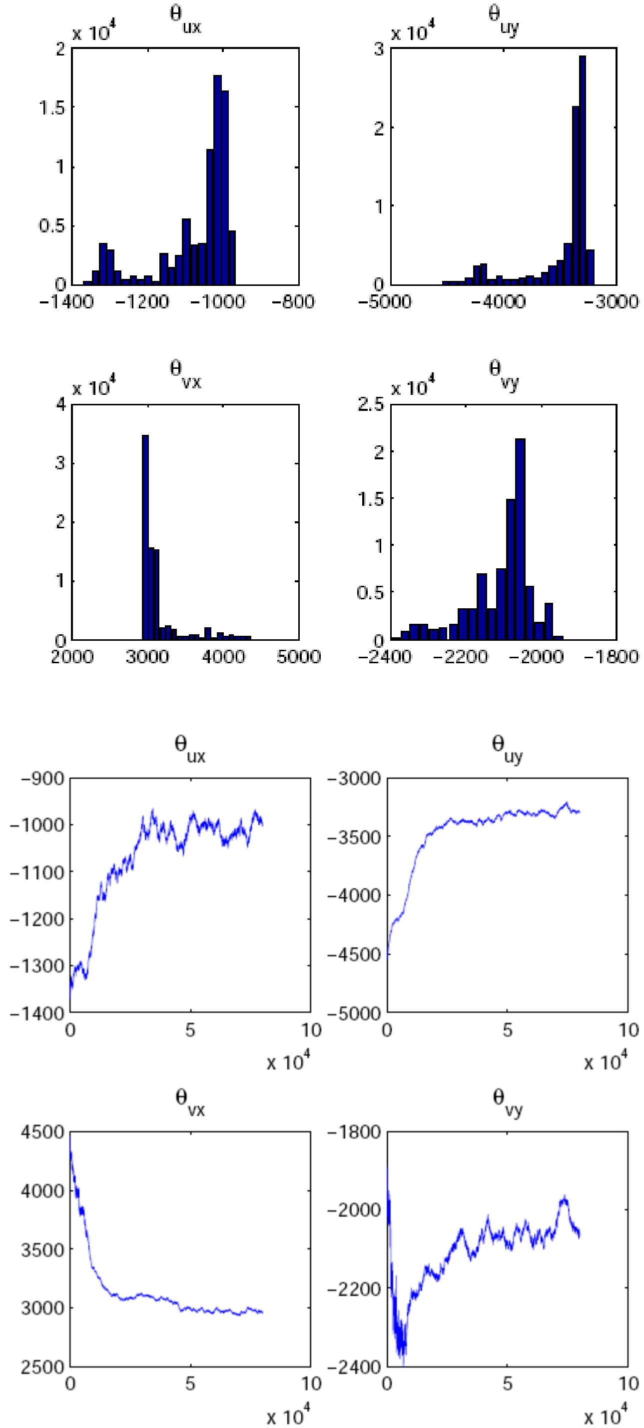
$$\begin{aligned} A &= K_s'K_s/\sigma_s^2 + K_a'K_a/\sigma_a^2 + I\sigma_u^2 \\ B &= S_t'K_s/\sigma_s^2 \cdot A_t'K_a/\sigma_a^2 \cdot \\ &\quad \theta_{ux}D_xP_t + \theta_{uy}D_yP_t + \epsilon_u \end{aligned}$$

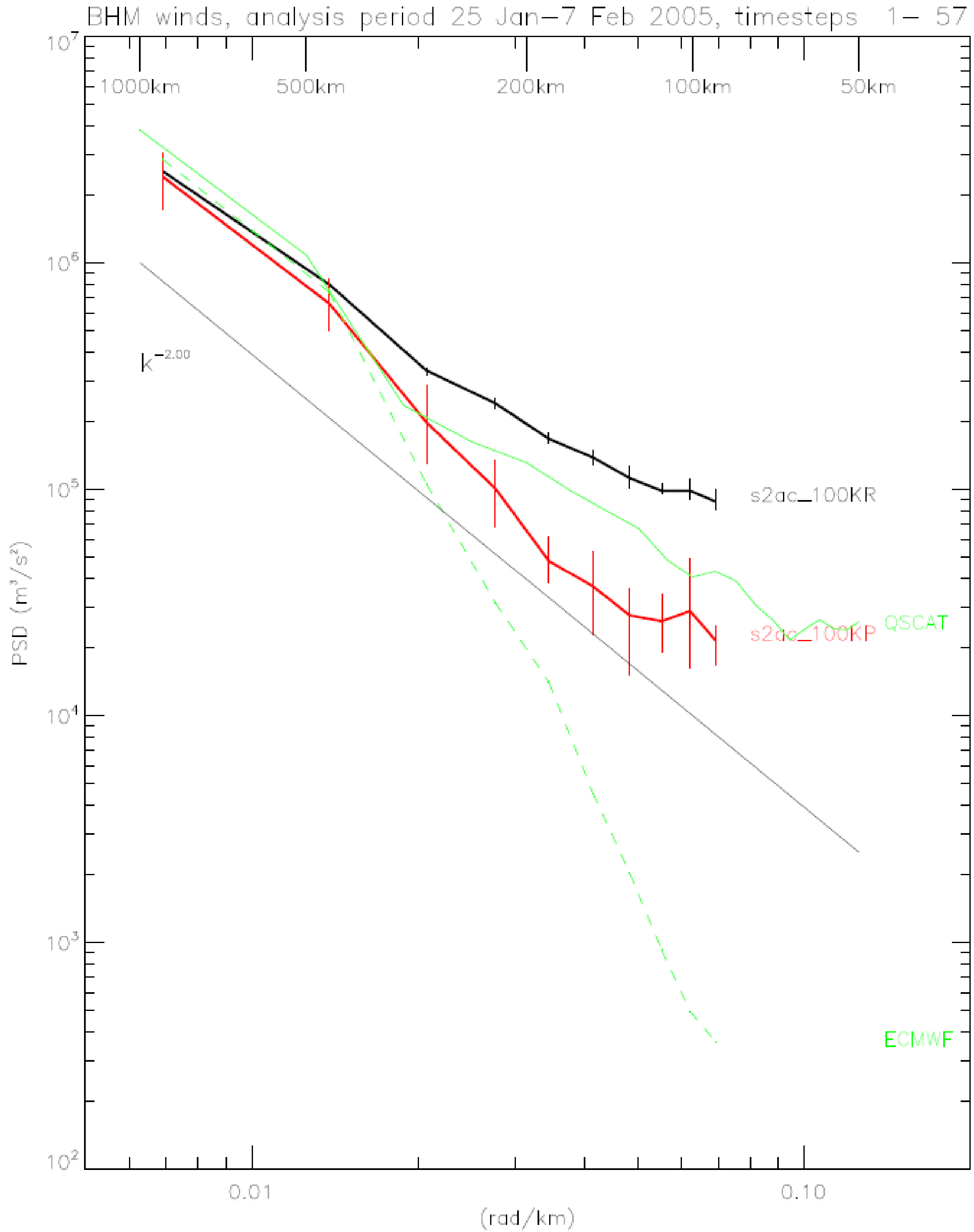
Variance

Similarly for v, P, parameters .....

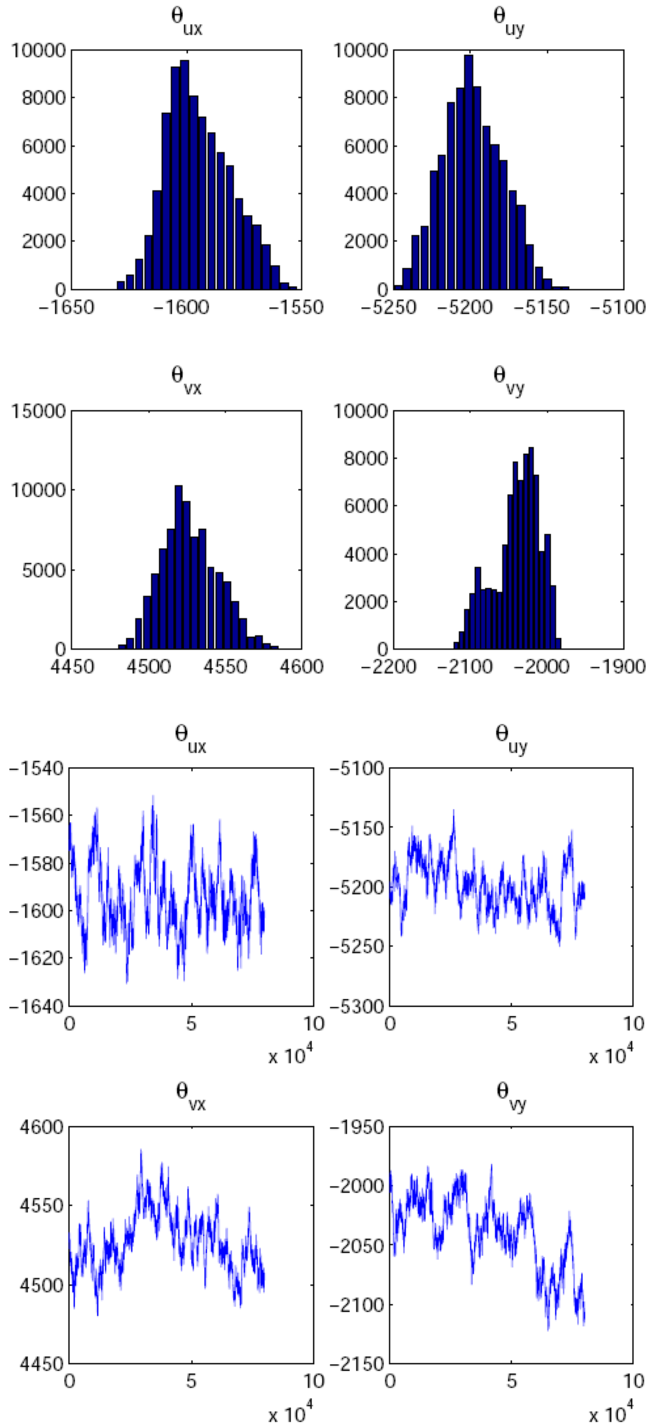

24-Jan-2005 Time Step: 3
45°N
40°N
35°N
30°N
5°W
0°
5°E
10°E
15°E
20°E
10 m s-1

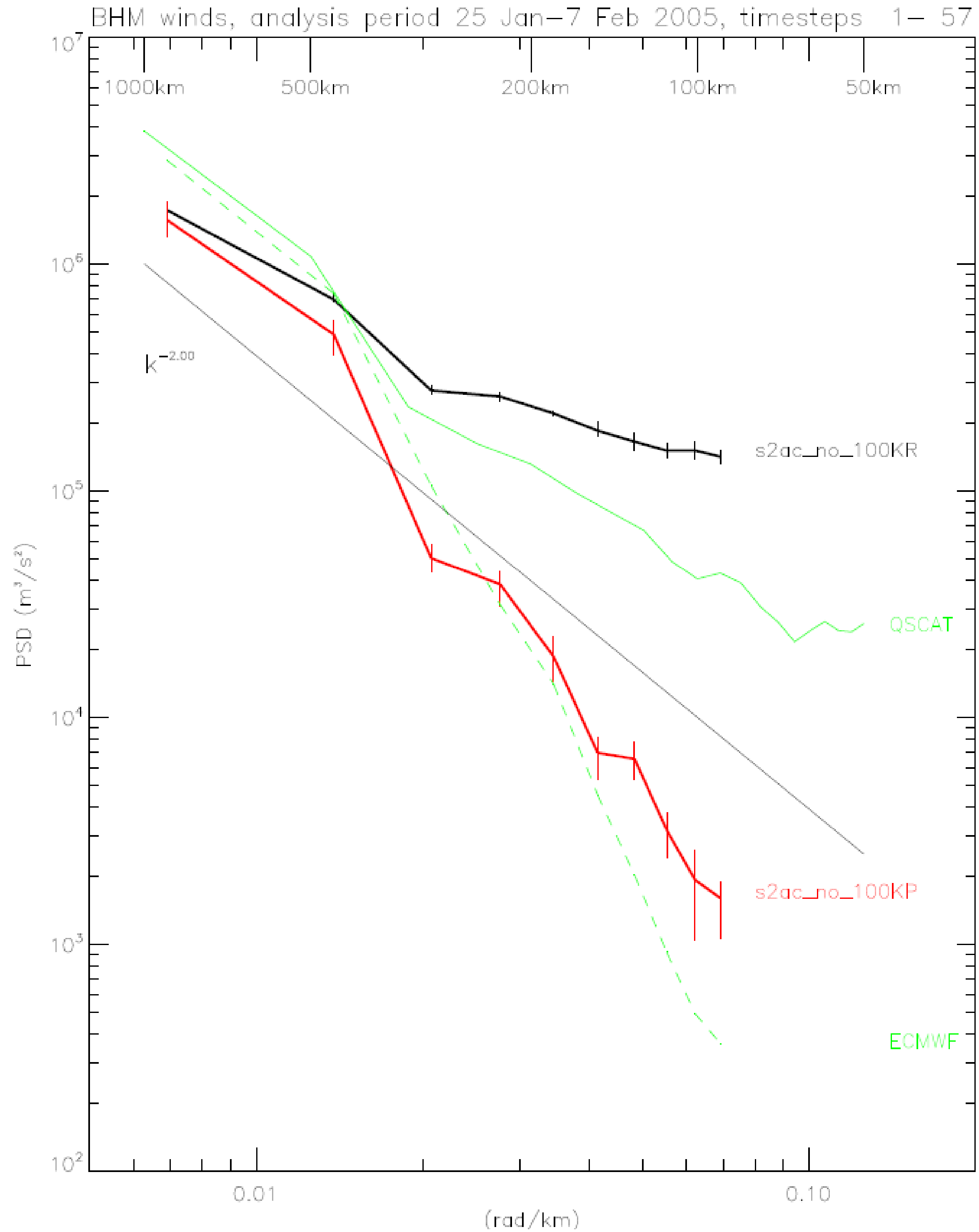
Posterior Distributions: QuikSCAT Data Stage Included

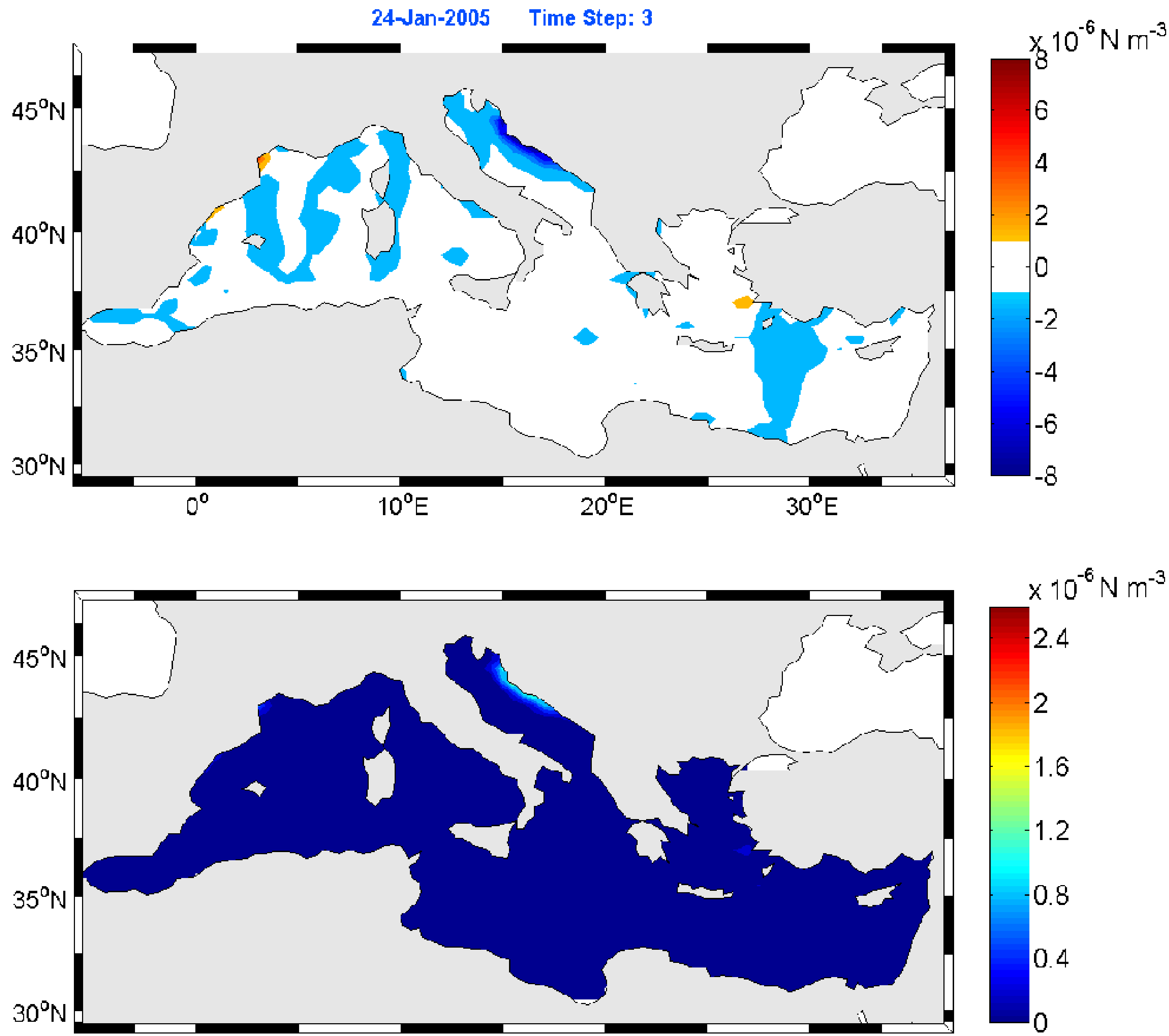




Posterior Distributions: QuikSCAT Data Stage ***Not*** Included






24-Jan-2005 Time Step: 3
45°N
40°N
35°N
30°N
0°
10°E
20°E
30°E
x 10^-6 N m^-3
8
6
4
2
0
-2
-4
-6
-8
45°N
40°N
35°N
30°N
x 10^-6 N m^-3
2.4
2
1.6
1.2
0.8
0.4
0

Progress and Plans

- *MFS-Wind-BHM implemented in Mediterranean Forecast System*
  generates physically realistic spread in forecast initial conditions
  spread focused on ocean mesoscale eddies
  temporally, regionally specific

- *MFS-Wind-BHM critically sensitive to QuikSCAT Data Stage*
  parameter posterior distributions more certain (modal) in QuikSCAT case
  realizations, posterior mean mimic power-law spectral behavior (vs. WN)

- *Physical Interpretability of Parameter Distributions*
  validation experiments; A1 and geostrophic data stage
  tradeoff: data stage volume vs. local, temporal specificity

- *Multi-variate Data Stage Inputs*
  extend multi-platform data stages (i.e. MFS-Wind-BHM)
  build parameterizations with uncertainty estimates

EXTRAS

A2: "Rayleigh Friction Model" (Linear Planetary Boundary Layer Equations)

continuous

$$\frac{\partial u}{\partial t} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \gamma u$$
$$\frac{\partial v}{\partial t} + fv = -\frac{1}{\rho}\frac{\partial p}{\partial y} - \gamma v$$

solve for u,v; substitute;
neglect second order time derivative

discrete

$$V_t = \left[1 + \frac{2\gamma}{f^2\Delta} + \frac{\gamma^2}{f^2}\right]^{-1} \left[(\frac{2\gamma}{f^2\Delta})V_{t-1} + (\frac{1}{f})D_xP_t + (-\frac{1}{f^2\Delta} - \frac{\gamma}{f^2})D_yP_t + (\frac{1}{f^2\Delta})D_yP_{t-1}\right]$$

$$U_t = \left[1 + \frac{2\gamma}{f^2\Delta} + \frac{\gamma^2}{f^2}\right]^{-1} \left[(\frac{2\gamma}{f^2\Delta})U_{t-1} + (-\frac{1}{f})D_yP_t + (-\frac{1}{f^2\Delta} - \frac{\gamma}{f^2})D_xP_t + (\frac{1}{f^2\Delta})D_xP_{t-1}\right]$$

(pre)stochastic

$$V_t = -L_{v|v(1)}V_{t-1} + c_{v|p_x}D_xP_t + c_{v|p_y}D_yP_t + c_{v|p_y(1)}D_yP_{t-1} + \epsilon$$

$$U_t = -L_{u|u(1)}U_{t-1} - c_{u|p_y}D_yP_t + c_{u|p_x}D_xP_t + c_{u|p_x(1)}D_xP_{t-1} + \epsilon$$

A2: Full Conditional for Surface Wind Component (*U*)

$$[U|\cdot] \;\propto\; [S_t^u|U_t,\theta_s][A_t^u|U_t,\theta_a]$$

Data Stage Distributions

$$[U_t|U_{t-1},P_t,P_{t-1},\ldots]$$

Process Model Stage Distributions

$$[U_{t+1}|U_t,P_{t+1},P_t,\ldots]$$

$$[U_t|\cdot] \sim N(A^{-1}B, A)$$

Gaussian Assumption
(less restrictive for *conditional* distributions)

$$A \;=\; K_s'K_s/\sigma_s^2 + K_a'K_a/\sigma_a^2 + I\sigma_u^2 + L_{u|u(1)}^2 I/\sigma_u^2$$

variance

$$\begin{aligned} B \;=\; & S_t'K_s/\sigma_s^2 A_t'K_a/\sigma_a^2 \\ & (-L_{u|u(1)}U_{t-1}' - c_{u|p_y}P_t'D_y' + c_{u|p_x}P_tD_x' + c_{u|p_x(1)}P_{t-1}'D_x' \\ & -L_{u|u(1)}U_{t+1}' - L_{u|u(1)}c_{u|p_y}P_{t+1}'D_y' + L_{u|u(1)}c_{u|p_x}P_{t+1}'D_x' \\ & +L_{u|u(1)}c_{u|p_x(1)}P_{t+1}'D_x')/\sigma_u^2 \end{aligned}$$

$$U_t = \left[f^2 + \gamma^2 + 2\frac{\gamma}{\Delta} \right]^{-1} \left\{ -\frac{f}{\rho_0} D_y P_t - \frac{1}{\rho_0} \left[\frac{1}{\Delta} + \gamma \right] D_x P_t + 2\frac{\gamma}{\Delta} U_{t-1} + \frac{1}{\rho_0 \Delta} D_x P_{t-1} \right\}$$

$$V_t = \left[f^2 + \gamma^2 + 2\frac{\gamma}{\Delta} \right]^{-1} \left\{ +\frac{f}{\rho_0} D_x P_t - \frac{1}{\rho_0} \left[\frac{1}{\Delta} + \gamma \right] D_y P_t + 2\frac{\gamma}{\Delta} V_{t-1} + \frac{1}{\rho_0 \Delta} D_y P_{t-1} \right\}$$

$$U_t = a_1 D_y P_t + a_2 D_x P_t + a_3 U_{t-1} + a_4 D_x P_{t-1} + \varepsilon_u$$

$$V_t = b_1 \, D_x P_t + b_2 D_y P_t + b_3 V_{t-1} + b_4 D_y P_{t-1} + \varepsilon_v$$

$$a_1 \approx N(-\frac{f}{\rho_0 \left(f^2 + \gamma^2 + 2\frac{\gamma}{\Delta} \right)}, \sigma_{a1}^2)$$

$$b_1 \approx N(\frac{f}{\rho_0 \left(f^2 + \gamma^2 + 2\frac{\gamma}{\Delta} \right)}, \sigma_{b1}^2)$$

$$a_2 \approx N(-\frac{1}{\rho_0 \left(f^2 + \gamma^2 + 2\frac{\gamma}{\Delta} \right)} \left[\frac{1}{\Delta} + \gamma \right], \sigma_{a2}^2)$$

$$b_2 \approx N(-\frac{1}{\rho_0 \left(f^2 + \gamma^2 + 2\frac{\gamma}{\Delta} \right)} \left[\frac{1}{\Delta} + \gamma \right], \sigma_{b2}^2)$$

$$a_3 \approx N(\frac{2\gamma}{\rho_0 \Delta \left(f^2 + \gamma^2 + 2\frac{\gamma}{\Delta} \right)}, \sigma_{a3}^2)$$

$$b_3 \approx N(\frac{2\gamma}{\rho_0 \Delta \left(f^2 + \gamma^2 + 2\frac{\gamma}{\Delta} \right)}, \sigma_{b3}^2)$$

$$a_4 \approx N(\frac{1}{\rho_0 \Delta \left(f^2 + \gamma^2 + 2\frac{\gamma}{\Delta} \right)}, \sigma_{a4}^2)$$

$$b_4 \approx N(\frac{1}{\rho_0 \Delta \left(f^2 + \gamma^2 + 2\frac{\gamma}{\Delta} \right)}, \sigma_{b4}^2)$$

Rayleigh Friction Equations

$$\frac{\partial u}{\partial t} - fv = -\frac{1}{\rho_o}\frac{\partial p}{\partial x} - \gamma u$$
$$\frac{\partial v}{\partial t} + fu = -\frac{1}{\rho_o}\frac{\partial p}{\partial y} - \gamma v$$

Scaling

$$(x, y) \sim L(x', y') \qquad (u, v) \sim U(u', v')$$

$$p \sim \rho_o f_o U L p'$$

$$t \sim \tfrac{L}{U} \qquad \gamma \sim \gamma_o \gamma'$$

$$f = f_o + \beta y \qquad f \sim f_o f'$$

$$f' = (1 + \tfrac{\beta y}{f_o})$$

$$\beta_o \sim \tfrac{U}{L^2}\beta' \qquad f' = [1 + \tfrac{\beta_o U L^2}{U f_o L}(y' - y_o')]$$

Define two small parameters: $\epsilon_1 = \dfrac{U}{f_o L}$ Rossby No. ; and $\epsilon_2 = \dfrac{\gamma_o}{f_o}$ $\dfrac{\text{Friction timescale}}{\text{Rotation timescale}}$

Scaled RFE:

$$\epsilon_1 \frac{\partial u}{\partial t} - (1 + \beta\epsilon_1 y)v = -\frac{\partial p}{\partial x} - \epsilon_2 u$$
$$\epsilon_1 \frac{\partial v}{\partial t} + (1 + \beta\epsilon_1 y)u = -\frac{\partial p}{\partial y} - \epsilon_2 v$$

Expansion:

Let $\epsilon_1 = \epsilon_2 = \epsilon$, and let

$$u = u_o + \epsilon u_1 + \epsilon^2 u_2 + ...$$
$$v = v_o + \epsilon v_1 + \epsilon^2 v_2 + ...$$
$$p = p_o + \epsilon p_1 + \epsilon^2 p_2 + ...$$

$O(0)$:

$$v_o = \frac{\partial p_o}{\partial x}$$
$$u_o = -\frac{\partial p_o}{\partial y}$$

$O(\epsilon)$:

$$\frac{\partial u_o}{\partial t} - v_1 - \beta y v_o = -\frac{\partial p_1}{\partial x} - u_o$$
$$\frac{\partial v_o}{\partial t} + u_1 - \beta y u_o = -\frac{\partial p_1}{\partial y} - v_o$$

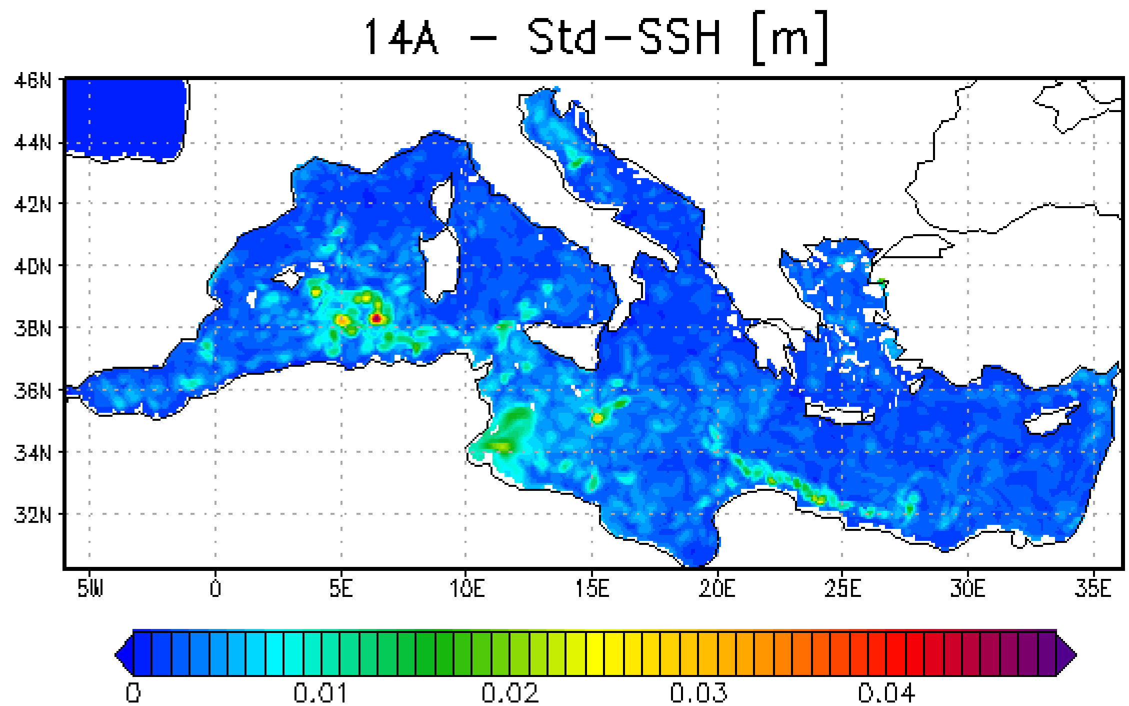
$$\frac{\partial}{\partial t}\left[\frac{\partial v_o}{\partial x} - \frac{\partial u_o}{\partial y}\right] + \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} + \beta v_o = -\frac{\partial v_o}{\partial x} + \frac{\partial u_o}{\partial y}$$

Recovers Barotropic Vorticity Eqn:

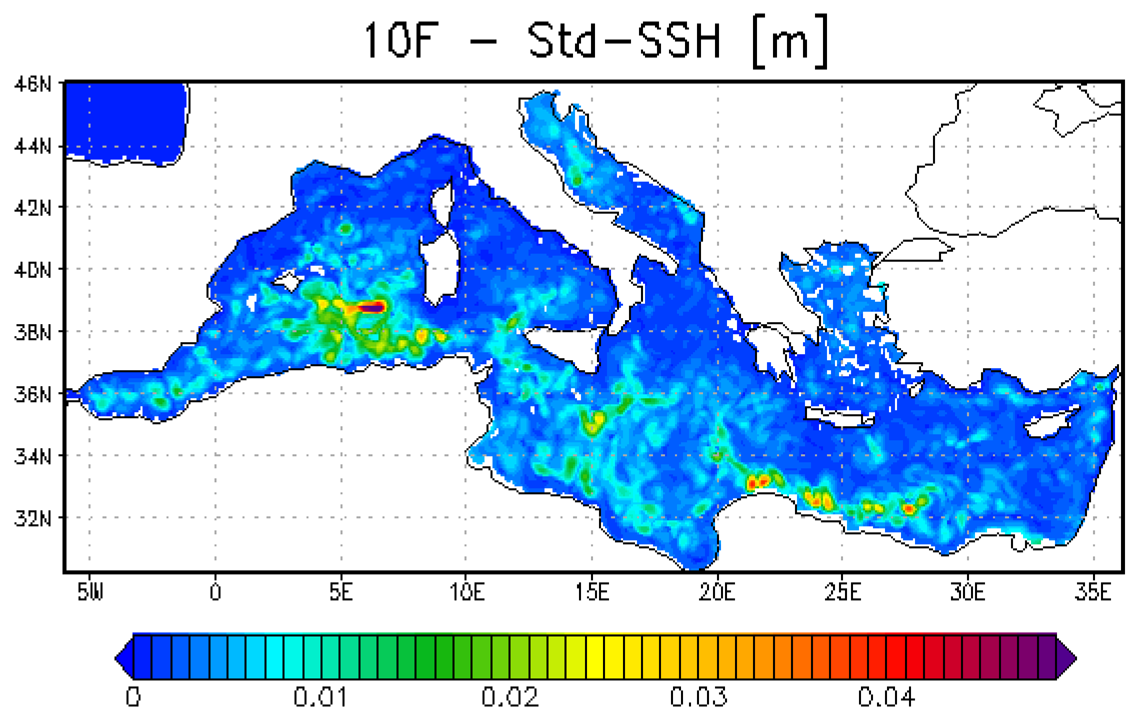
$$\frac{\partial}{\partial t}\left[{\nabla_H}^2 p_o\right] + \beta\frac{\partial p_o}{\partial x} = -\vec{\nabla}_H \cdot \vec{u_1} - \nabla^2 p_o$$

Initial Condition and Forecast Spread in response to MFS-Wind-BHM

Ensemble Initial condition
SSH standard deviation
10 members



Ensemble Foercast, day 10
SSH standard deviation
10 members



Response to v8_CKW

Forecast Uncertainty Localization: *Mesoscale Eddies, Persistent Hot Spots*

Std Dev (SSH)
MFS-Wind BHM
Forecast *day 10F*
18 Feb 2005

Long-Term Avg
MFS Forecast Misfits
vs. Altimeter
Sep 2004 – Mar 2005